Quadratic Equations

Question1

If $x^2 - 4ax + 5 + a > 0$ for all $x \in R$ whenever $a \in (\alpha, \beta)$, then $4\beta + \alpha =$

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Options:

A.

0

B.

4

C.

5

D.

8

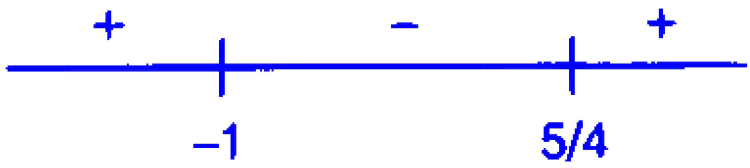
Answer: B

Solution:

$x^2 - 4ax + 5 + a > 0$

so, $D < 0$

$$\begin{aligned}
\Rightarrow \quad & (-4a)^2 - 4(1)(5 + a) < 0 \\
\Rightarrow \quad & 16a^2 - 20 - 4a < 0 \\
\Rightarrow \quad & 4a^2 - a - 5 < 0 \\
\Rightarrow \quad & 4a^2 - 5a + 4a - 5 < 0 \\
\Rightarrow \quad & a(4a - 5) + 1(4a - 5) < 0 \\
\Rightarrow \quad & (4a - 5)(a + 1) < 0
\end{aligned}$$



So, $-1 < a < \frac{3}{4}$ that means, $\alpha = -1, \beta = 5/4$

$4\beta + \alpha = 4 \times \frac{5}{4} - 1 = 4$

Question2

If α, β, γ are the roots of the equation $x^3 - 12x^2 + kx - 18 = 0$ and one of them is thrice the sum of the other two roots, then $\alpha^2 + \beta^2 + \gamma^2 - k =$

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Options:

A.

115

B.

41

C.

56

D.

57

Answer: D

Solution:

$\because x^3 - 12x^2 + kx - 18 = 0$

So, $\quad \alpha + \beta + \gamma = -(-12)/1 = 12 \quad \ldots (i)$

$\alpha\beta + \beta\gamma + \gamma\alpha = k \quad \ldots (ii)$

and $\alpha\beta\gamma = -(-18)/1 = 18$ $\quad \ldots(iii)$

Let, $\alpha = 3(\beta + \gamma)$ $\quad \ldots(iv)$

Using Eq. (i), we get

$$3(\beta + \gamma) + (\beta + \gamma) = 12$$
$$\Rightarrow \quad \beta + \gamma = 3$$

So, $\alpha = 3 \times 3 = 9$ $\quad \ldots(v)$

and $9\beta\gamma = 18$

$\Rightarrow \quad \beta\gamma = 2$ $\quad \ldots(vi)$

From Eqs. (v) and (vi), we get

$\beta = 1, \gamma = 2$

$$Therefore, k = 9 \times 1 + 1 \times 2 + 2 \times 9$$
$$= 9 + 2 + 18 = 29$$

Hence, $\alpha^2 + \beta^2 + \gamma^2 - k$

$$= 9^2 + 1^2 + 2^2 - 29$$
$$= 81 + 1 + 4 - 29 = 57$$

Question3

The polynomial equation of degree 5 whose roots are the roots of the equation $x^5 - 3x^4 - x^3 + 11x^2 - 12x + 4 = 0$ each increased by 2 , is

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Options:

A.

$x^5 - 13x^4 + 63x^3 - 135x^2 - 108x = 0$

B.

$x^5 - 13x^4 + 63x^3 + 135x^2 + 108x = 0$

C.

$x^5 - 13x^4 + 63x^3 - 135x^2 + 108x = 0$

D.

$x^5-13x^4-63x^3-135x^2-108x=0$

Answer: C

Solution:

Let

$P(x)=x^5-3x^4-x^3+11x^2-12x+4$

When each roots of the equation increased by 2 , then

$P(x) \rightarrow P(x-2)$

So, $P(x-2)=(x-2)^5-3(x-2)^4$

$$\begin{gathered}
-(x-2)^3+11(x-2)^2-12(x-2)+4 \\
=x^5-10x^4+40x^3-80x^2+80x-32 \\
-3\left(x^4-8x^3+24x^2-32x+16\right) \\
-\left(x^3-6x^2+12x-8\right)+11\left(x^2-4x+4\right) \\
-12(x-2)+4 \\
=x^5-13x^4+63x^3-135x^2+108x
\end{gathered}$$

Hence, the required equation

$x^5-13x^4+63x^3-135x^2+108x=0$

Question4

If the area of a square is 575 square units, then the approximate value of its side is

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Options:

A.

23.9792

B.

23.7992

C.

23.8687

D.

23.7868

Answer: A

Solution:

Let the side length as ' a '

So, $a^2 = 575$

$\Rightarrow a = \sqrt{575}$

and $529 < 575 < 576$

So, $\sqrt{575} = 23 + \frac{575-529}{576-529} = 23 + \frac{46}{47}$

$= 23.9792$

Question5

If the difference of the roots of the equation $x^2 - 7x + 10 = 0$ is same as the difference of the roots of the equation $x^2 - 17x + k = 0$, then a divisor of k is $x^2 - 7x + 10 = 0$

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Options:

A.

14

B.

17

C.

6

D.

15

Answer: A

Solution:

$x^2 - 7x + 10 = 0$

$\because \quad a = 1$

$\therefore$ Difference of roots $= \dfrac{\sqrt{D}}{|a|}$

$= \dfrac{\sqrt{49 - 40}}{1} = \sqrt{9} = 3$

For quadratic equation

$x^2 - 17x + k = 0$

Difference of roots $= \sqrt{289 - 4k}$

According to the questions,

$3 = \sqrt{289 - 4k}$

Squaring both sides, we get

$\Rightarrow \quad 9 = 289 - 4k \Rightarrow 4k = 280$
$\Rightarrow \quad k = 70$

$\therefore$ Divisor of $k = 14$

Question6

The product of all the real roots of the equation $|x|^2 - 5|x| + 6 = 0$

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Options:

A.

25

B.

36

C.

4

D.

16

Answer: B

Solution:

$$
\begin{aligned}
&\therefore |x|^2-5|x|+6=0\\
&\Rightarrow \quad |x|^2-2|x|-3|x|+6=0\\
&\Rightarrow \quad |x|(|x|-2)-3(|x|-2)=0\\
&\Rightarrow \quad (|x|-2)(|x|-3)=0\\
&\Rightarrow \quad |x|=2, |x|=3\\
&\Rightarrow \quad x=\pm 2, x=\pm 3\\
&\therefore \text{ Product of all roots } =2\times(-2)\times 3(-3)\\
&\quad =4\times 9=36
\end{aligned}
$$

Question7

If α, β and γ are the roots of the equation $5x^3-4x^2+3x-2=0$, then $\alpha^3+\beta^3+\gamma^3=$

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Options:

A.

$\frac{17}{25}$

B.

$\frac{394}{125}$

C.

$\frac{34}{125}$

D.

$\frac{34}{25}$

Answer: C

Solution:

Let the given equation be $P(x) = 5x^3 - 4x^2 + 3x - 2 = 0$.

The roots of the equation are α, β, γ.

According to Vieta's formulas, we have:

1. Sum of the roots: $\alpha + \beta + \gamma = -(\frac{-4}{5}) = \frac{4}{5}$.
2. Sum of the products of the roots taken two at a time: $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{3}{5}$.
3. Product of the roots: $\alpha\beta\gamma = -(\frac{-2}{5}) = \frac{2}{5}$.

We want to find the value of $\alpha^3 + \beta^3 + \gamma^3$.

Method 1: Using the identity for sum of cubes.

We know the identity:

$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - (\alpha\beta + \beta\gamma + \gamma\alpha))$.

First, let's find $\alpha^2 + \beta^2 + \gamma^2$:

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$.

Substitute the values from Vieta's formulas:

$\alpha^2 + \beta^2 + \gamma^2 = \left(\frac{4}{5}\right)^2 - 2\left(\frac{3}{5}\right)$

$= \frac{16}{25} - \frac{6}{5}$

$= \frac{16}{25} - \frac{30}{25}$

$= -\frac{14}{25}$.

Now, substitute all the required values into the sum of cubes identity:

$\alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - (\alpha\beta + \beta\gamma + \gamma\alpha)) + 3\alpha\beta\gamma$.

$\alpha^3 + \beta^3 + \gamma^3 = \left(\frac{4}{5}\right)\left(-\frac{14}{25} - \frac{3}{5}\right) + 3\left(\frac{2}{5}\right)$.

$\alpha^3 + \beta^3 + \gamma^3 = \left(\frac{4}{5}\right)\left(-\frac{14}{25} - \frac{15}{25}\right) + \frac{6}{5}$.

$\alpha^3 + \beta^3 + \gamma^3 = \left(\frac{4}{5}\right)\left(-\frac{29}{25}\right) + \frac{6}{5}$.

$\alpha^3 + \beta^3 + \gamma^3 = -\frac{116}{125} + \frac{6}{5}$.

To sum these fractions, find a common denominator, which is 125:

$\alpha^3 + \beta^3 + \gamma^3 = -\frac{116}{125} + \frac{6\times25}{5\times25}$.

$\alpha^3 + \beta^3 + \gamma^3 = -\frac{116}{125} + \frac{150}{125}$.

$\alpha^3 + \beta^3 + \gamma^3 = \frac{150-116}{125}$.

$\alpha^3 + \beta^3 + \gamma^3 = \frac{34}{125}$.

Method 2: Using Newton's Sums.

Since α, β, γ are roots of $5x^3 - 4x^2 + 3x - 2 = 0$, they satisfy the equation:

$5\alpha^3 - 4\alpha^2 + 3\alpha - 2 = 0$

$5\beta^3 - 4\beta^2 + 3\beta - 2 = 0$

$5\gamma^3 - 4\gamma^2 + 3\gamma - 2 = 0$

Summing these three equations:

$5(\alpha^3 + \beta^3 + \gamma^3) - 4(\alpha^2 + \beta^2 + \gamma^2) + 3(\alpha + \beta + \gamma) - 2(1 + 1 + 1) = 0.$

$5(\alpha^3 + \beta^3 + \gamma^3) - 4(\alpha^2 + \beta^2 + \gamma^2) + 3(\alpha + \beta + \gamma) - 6 = 0.$

Let $p_1 = \alpha + \beta + \gamma = \frac{4}{5}$.

Let $p_2 = \alpha^2 + \beta^2 + \gamma^2$. We calculated this above: $p_2 = -\frac{14}{25}$.

Let $p_3 = \alpha^3 + \beta^3 + \gamma^3$.

Substitute p_1 and p_2 into the sum equation:

$5p_3 - 4\left(-\frac{14}{25}\right) + 3\left(\frac{4}{5}\right) - 6 = 0.$

$5p_3 + \frac{56}{25} + \frac{12}{5} - 6 = 0.$

To combine the constant terms, find a common denominator (25):

$5p_3 + \frac{56}{25} + \frac{12\times5}{5\times5} - \frac{6\times25}{25} = 0.$

$5p_3 + \frac{56}{25} + \frac{60}{25} - \frac{150}{25} = 0.$

$5p_3 + \frac{56+60-150}{25} = 0.$

$5p_3 + \frac{116-150}{25} = 0.$

$5p_3 - \frac{34}{25} = 0.$

$5p_3 = \frac{34}{25}.$

$p_3 = \frac{34}{25\times5}.$

$p_3 = \frac{34}{125}.$

Both methods yield the same result.

The final answer is $\boxed{\frac{34}{125}}$.

Question8

After the roots of the equation $6x^3 + 7x^2 - 4x - 2 = 0$ are diminished by h, if the transformed equation does not contain x term, then the product of all the possible value of h is

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Options:

A.

$1/3$

B.

$-2/3$

C.

$-2/9$

D.

$7/3$

Answer: C

Solution:

Given, $6x^3 + 7x^2 - 4x - 2 = 0$

Let x be the root and new root $x' = x - n$

$x = x' + n$

$6(x' + n)^3 + 7(x' + n)^2 - 4(x' + n) - 2 = 0$

Coefficient of x' :

$$18n^2 + 14n - 4 = 0$$
$$\Rightarrow \quad 9n^2 + 7n - 2 = 0$$
$$\Rightarrow \quad 9n^2 + 9n - 2n - 2 = 0$$

$$\Rightarrow \quad 9n(n+1) - 2(n+1) = 0 \Rightarrow n = -1, \frac{2}{9}$$

Product of all possible values of $n = -2/9$

Question9

The number of distinct quadratic equations $ax^2 + bx + c = 0$ with unequal real roots that can be formed by choosing the coefficients $a, b, c (a \neq b \neq c)$ from the set $\{0, 1, 2, 4\}$ is

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Options:

A.

4

B.

6

C.

5

D.

12

Answer: B

Solution:

$$ax^2 + bx + c = 0, a \neq b \neq c$$
$$a, b, c \in \{0, 1, 2, 4\}, a \neq 0$$

For unequal real roots

$D > 0 \Rightarrow b^2 > 4ac$

Let $b = 4, 4ac < 16$

$$\begin{aligned} &\Rightarrow ac < 4 \\ &c = 0, a = 1, 2 \rightarrow 2 \\ &b = 2, 4ac < 4 \Rightarrow ac < 1 \\ &c = 0, a = 1, 4 \rightarrow 2 \end{aligned}$$

$$b = 1, 4ac < 1 \Rightarrow ac < \frac{1}{4}$$

$$c = 0, a = 2, 4 \rightarrow 2$$
$$\therefore \text{ Total cases } = 2 + 2 + 2 = 6$$

Question10

The number of solutions of the equation $\sqrt{3x^2+x+5}=x-3$ is

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Options:

A.

2

B.

1

C.

0

D.

4

Answer: C

Solution:

Given, equation is

$\sqrt{3x^2+x+5}=x-3$

Here, $3x^2+x+5 \geq 0$ (since square root is non-negative)

$\Rightarrow \quad x-3 \geq 0$

$\Rightarrow \quad x \geq 3$

Now, squaring to both sides of given equation, we get

$$\left(\sqrt{3x^2+x+5}\right)^2=(x-3)^2$$
$$\Rightarrow \quad 3x^2+x+5=x^2-6x+9$$
$$\Rightarrow \quad 2x^2+7x-4=0$$

Using quadratic formula,

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot (-4)}}{2 \cdot 2}$$
$$= \frac{-7 \pm \sqrt{49 + 32}}{4} = \frac{-7 \pm 9}{4}$$
$$\Rightarrow \quad x = \frac{2}{4} \text{ and } x = \frac{-16}{4}$$
$$\Rightarrow \quad x = \frac{1}{2} \text{ and } x = -4$$

Since domain, $x \geq 3$, so $x = -4$

And $x = \frac{1}{2}$ are not possible.

$\therefore$ There is zero solution.

Question11

The set of all real values of x for which $\frac{x^2-1}{(x-4)(x-3)} \geq 1$ is

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Options:

A.

$[-1, 1] \cup (3, 4)$

B.

$\left[\frac{13}{7}, 3\right) \cup (4, \infty)$

C.

$\left(-\infty, \frac{13}{7}\right] \cup (3, 4)$

D.

$R - [3, 4]$

Answer: B

Solution:

Given, $\frac{x^2-1}{(x-4)(x-3)} \geq 1$

$$\Rightarrow \quad \frac{x^2-1}{(x-4)(x-3)}-1 \geq 0$$

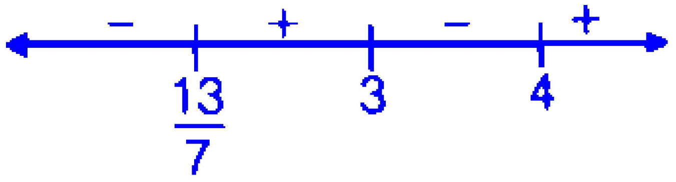
$$\Rightarrow \quad \frac{x^2-1-(x-4)(x-3)}{(x-4)(x-3)} \geq 0$$

Since, the inequality is $\frac{7x-13}{x^2-7x+12} \geq 0$

So, the critical points at $7x-13=0$

$\Rightarrow \quad x=\frac{13}{7}$

and at $x=4$ and $x=3$, the expression is undefined.



$\therefore x \in\left[\frac{13}{7}, 3\right) \cup(4, \infty)$

Question12

If α, β and γ are the roots of the equation $2x^3+3x^2-5x-7=0$, then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=$

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Options:

A.

$-\frac{17}{49}$

B.

$-\frac{23}{49}$

C.

$\frac{55}{49}$

D.

$\frac{67}{49}$

Answer: D

Solution:

Given, equation is

$2x^3+3x^2-5x-7=0$

Since, α, β and γ are the roots of the given equation.

$\therefore$ Sum of roots $=\alpha+\beta+\gamma=\frac{-3}{2}$

Sum of products of roots

$=\alpha\beta+\beta\gamma+\gamma\alpha=-\frac{5}{2}$

Products of roots $=\alpha\beta\gamma=-\left(\frac{-7}{2}\right)=\frac{7}{2}$

$$\begin{aligned}\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} &= \frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2} \\ &= \frac{\left(\beta\gamma^2\right)+(\gamma\alpha)^2+(\alpha\beta)^2}{(\alpha\beta\gamma)^2}\end{aligned}$$

$$\begin{aligned} &= \frac{(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2} \\ &= \frac{\left(\frac{-5}{2}\right)^2-2\left(\frac{7}{2}\right)\left(\frac{-3}{2}\right)}{\left(\frac{7}{2}\right)^2} \\ &= \frac{\frac{25}{4}+\frac{21}{2}}{\frac{49}{4}}=\frac{\frac{67}{4}}{\frac{49}{4}} \\ &= \frac{67}{4}\times\frac{4}{49}=\frac{67}{49} \\ \therefore \quad & \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{67}{49}\end{aligned}$$

Question13

Two roots of the equation, $ax^4+bx^3+cx^2+dx+e=0$ are positive and equal. If the product of the other two real roots is 1 , then

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Options:

A.

$be^2=a^2d$

B.

$3e + \frac{2b\sqrt{e}}{\sqrt{a}} + c = a$

C.

$e + 2b\sqrt{e} + 3c = a\sqrt{a}$

D.

$b^2 e = ad^2$

Answer: B

Solution:

Given equation

$ax^4 + bx^3 + cx^2 + dx + e = 0$

Let $\alpha, \alpha, \beta, \frac{1}{\beta}$ be root of equation

$$\therefore \quad \alpha + \alpha + \beta + \frac{1}{\beta} = -b/a$$

$$\Rightarrow \quad \left(\beta + \frac{1}{\beta}\right) = -\frac{b}{a} - 2\alpha$$

$$\Rightarrow \quad \alpha \cdot \alpha \cdot \beta \cdot \frac{1}{\beta} = \frac{e}{a} \Rightarrow \alpha^2 = e/a$$

$$\Rightarrow \quad \alpha^2 + 2(\alpha\beta) + \frac{2\alpha}{\beta} + 1 = c/a$$

$$\alpha^2 + 2\alpha\left(\beta + \frac{1}{\beta}\right) + 1 = c/a$$

$$\Rightarrow \quad \alpha^2 + 2\alpha\left(-\frac{b}{a} - 2\alpha\right) + 1 = c/a$$

$$\Rightarrow \quad -3\alpha^2 - \frac{2\alpha b}{a} + 1 = c/a$$

$$\Rightarrow \quad \frac{c}{a} + 3\alpha^2 + \frac{2\alpha b}{a} = 1$$

$$\Rightarrow \quad 3\left(\frac{e}{a}\right) + 2\sqrt{\frac{e}{a}} \times \frac{b}{c} + \frac{c}{a} = 1$$

$$\Rightarrow \quad 3e + 2b\sqrt{\frac{e}{a}} + c = a$$

Question14

Let $(a-3)x^2 + 12x + (a+6) > 0, \forall x \in R$ and $a \in (\ell, \infty)$. If a is the least positive integral value of a, then the roots of $(\alpha - 3)x^2 + 12x + (\ell + 2) = 0$ are

Options:

A.

1,2

B.

2,3

C.

$-1, -2$

D.

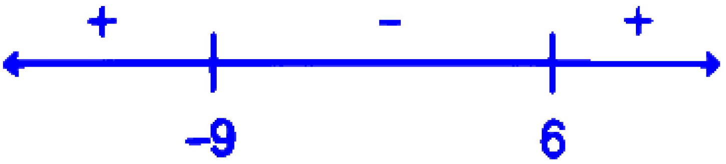
$-2, -3$

Answer: C

Solution:

Let $f(x) = (a-3)x^2 + 12x + (a+6)$ to quadratic always be positive

$$\begin{aligned} & a-3>0 \\ \Rightarrow \quad & a>3 \\ \text{and} \quad & D=(12)^2-4(a-3)(a+6)<0 \\ & =144-4\left(a^2+3a-18\right)<0 \\ \Rightarrow \quad & a^2+3a-54>0 \\ \Rightarrow \quad & a^2+9a-6a-54>0 \\ \Rightarrow \quad & (a+9)(a-6)>0 \end{aligned}$$



$a \in (-\infty, -9) \cup (6, \infty)$

Combine with $a > 3$

$\Rightarrow$ We take $a \in (6, \infty)$

Least positive integeral value

$\alpha = 7, l = 6$

put these values

$$
\begin{aligned}
& (\alpha-3)x^2+12x+(l+2) \\
& \quad =4x^2+12x+8=0 \\
\Rightarrow \quad & x^2+3x+2=0 \\
\Rightarrow \quad & (x+1)(x+2)=0
\end{aligned}
$$

Hence, $x=-1,-2$

Question15

If the roots of the equation $x^2+2ax+b=0$ are real, distinct and differ atmost by 2 m , then b lies in the interval

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Options:

A.

$\left(a^2, a^2+m^2\right)$

B.

$\left(a^2+m^2, a^2\right)$

C.

$\left[a^2, a^2+2m^2\right]$

D.

$\left[a^2-m^2, a^2\right)$

Answer: D

Solution:

$\because x^2+2ax+b=0$

$$
\begin{aligned}
\Rightarrow \quad x & =\frac{-2a \pm \sqrt{(2a)^2-4b}}{2} \\
& =-a \pm \sqrt{a^2-b}
\end{aligned}
$$

let,

$\alpha=-a+\sqrt{a^2-b}$
$\beta=-a-\sqrt{a^2-b}$

so, $|\alpha-\beta|=2\sqrt{a^2-b}$

given that $2\sqrt{a^2-b}\leq 2m$

$\Rightarrow \quad a^2-b\leq m^2$
$\Rightarrow \quad b\geq a^2-m^2 \qquad \ldots(i)$

Also, for real and distinct roots

$$D=4a^2-4b>0$$

$\Rightarrow \quad b<a^2 \qquad \ldots(ii)$

Hence, $a^2-m^2 \leq b

$\Rightarrow \quad b\in\left[a^2-m^2,a^2\right)$

Question16

The cubic equation whose roots are the squares of the roots of the equation $x^3-2x^2+3x-4=0$ is

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Options:

A.

$x^3+2x^2+7x-16=0$

B.

$x^3+2x^2-7x-16=0$

C.

$x^3-2x^2-7x+16=0$

D.

$x^3-2x^2+7x+16=0$

Answer: B

Solution:

$x^3 - 2x^2 + 3x - 4 = 0$
So, $\alpha + \beta + \gamma = 2\alpha\beta + \beta\gamma + \gamma\alpha = +3,$
$\alpha\beta\gamma = 4$
thus, $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2$
$- 2(\alpha\beta + \beta\gamma + \gamma\alpha)$
$= 4 - 2(3) = -2$
and $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$
$= (\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2$
$\because \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$

$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$
$= (3)^2 - 2(4)(2) = 9 - 16 = -7$

and $\alpha^2\beta^2\gamma^2 = (\alpha\beta\gamma)^2 = 4^2 = 16$

Hence, the cubic equation with roots $\alpha^2, \beta^2, \gamma^2$ is

$x^3 - \left(\alpha^2 + \beta^2 + \gamma^2\right)x^2 + \left(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2\right)x - \alpha^2\beta^2\gamma^2$
$= x^3 - (-2)x^2 + (-7)x - 16 \quad x - \alpha^2\beta^2\gamma^2$
$= x^3 + 2x^2 - 7x - 16 = 0$

Question17

If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then $\alpha^3 + \beta^3 + \gamma^3 =$

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Options:

A.

$p^3 - 3pq + r$

B.

$p^2 - 2pq + r$

C.

$3pq - 3r - p^3$

D.

$3pq + 3r + p^3$

Answer: C

Solution:

$$\because x^3 + px^2 + qx + r = 0$$
$$\alpha + \beta + \gamma = -p, \alpha\beta + \beta\gamma + \gamma\alpha = q,$$
$$\alpha\beta\gamma = -r$$

$$So, \; \alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta + \gamma)^3 - 3(\alpha + \beta + \gamma)(\alpha\beta + \beta\gamma + \gamma\alpha) + 3\alpha\beta\gamma$$
$$= (-p)^3 - 3(-p)(q) + 3(-r)$$
$$= -p^3 + 3pq - 3r$$

Question18

If α, β are the roots of the equation $x^2 + bx + c = 0$ satisfying the conditions $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 60$, then $3c + 2 =$

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Options:

A.

$2b$

B.

$3b$

C.

$-3b$

D.

$-2b$

Answer: C

Solution:

$\because \alpha+\beta=5 \Rightarrow \frac{-b}{1}=5 \Rightarrow b=-5 \ldots$ (i)

and $\alpha^3+\beta^3=60$

$\Rightarrow \quad (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=60$

$\Rightarrow \quad (5)^3-3\times c(5)=60$

$\Rightarrow \quad 125-15c=60$

$\Rightarrow \quad 15c=125-60=65$

$\Rightarrow \quad c=\frac{65}{15}=\frac{13}{3}$

$\therefore 3c+2=3\times\frac{13}{3}+2=13+2=15$

$=(-3)(-5)=-3b$ (from Eq. (i))

Question19

If α, β, γ are the roots of the equation,

$x^3+ax^2+bx+c=0$, then $(\alpha+\beta-2\gamma)$
$(\beta+\gamma-2\alpha)(\gamma+\alpha-2\beta)=$

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Options:

A.

$2a^3+9ab+27c$

B.

$2a^3+9ab-27c$

C.

$2a^3-9ab-27c$

D.

$2a^3-9ab+27c$

Answer: D

Solution:

Given, α, β and γ are roots of equation

$$x^3 + ax^2 + bx + c = 0$$
$$\alpha + \beta + \gamma = -a$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = b$$
$$\alpha\beta\gamma = -c$$

Now, $(\alpha + \beta - 2y)(\beta + \gamma - 2\alpha)(\gamma + \alpha - 2\beta)$

$$= (\alpha + \beta + \gamma - 3\gamma)(\alpha + \beta + \gamma - 3\alpha)(\alpha + \beta + \gamma - 3\beta)$$
$$= -(\alpha + 3\alpha)(a + 3\beta)(a + 3\gamma)$$

$$= -\left(a^3 + 3(\alpha + \beta + \gamma)a^2 + 9(\alpha\beta + \beta\gamma + \gamma\alpha)a + 27(\alpha\beta\gamma)\right.$$
$$= -\left(a^3 - 3a^3 + 9ab - 27c\right)$$
$$= 2a^3 - 9ab + 27c$$

Question20

If the sum of two roots of the equation $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ is zero, then the sum of the squares of the other two roots is

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Options:

A.

5

B.

10

C.

13

D.

25

Answer: B

Solution:

Given,

$$x^4+2x^3-7x^2-8x+12=0$$
$$\Rightarrow(x-1)(x-3)(x-2)(x+2)=0$$
$$\Rightarrow x=1,3,2-2$$

Since sum of two roots of Eq. (i) is zero.

So, $-2+2=0$

Hence, sum of the squares of the other two roots

$(1)^2+(3)^2=10$

Question21

$f(x)$ is a quadratic polynomial satisfying the condition $f(x)+f\left(\frac{1}{x}\right)=f(x) f\left(\frac{1}{x}\right)$. If $f(-1)=0$, then the range of f is

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Options:

A.

$[1, \infty)$

B.

$[-1,1]$

C.

$(-\infty, 1]$

D.

R

Answer: C

Solution:

$f(x)=a x^2+b x+c$

substitute $f(x)$ into the given equation

$$ax^2+bx+c+a\left(\frac{1}{x}\right)^2+b\left(\frac{1}{x}\right)+c$$
$$=\left(ax^2+bx+c\right)\left(a\left(\frac{1}{x}\right)^2+b\left(\frac{1}{x}\right)+c\right)$$
$$\Rightarrow ax^2+bx+c+\frac{a}{x^2}+\frac{b}{x}+c$$
$$=\left(ax^2+bx+c\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)$$
$$\Rightarrow ax^4+bx^3+cx^2+a+bx+cx^2$$
$$=\left(ax^2+bx+c\right)\left(a+bx+cx^2\right)$$
$$\Rightarrow ax^4+bx^3+2cx^2+bx+a$$
$$=a^2x^2+abx^3+acx^4+abx+b^2x^2+bcx^3+ac+bcx+c^2x^2$$

Compare coefficients : $a=ac$,

$b=ab+bc$
$2c=a^2+b^2+c^2, b=ab+bc, a=ac$

From $a=ac$, we have $c=1$ or $a=0$, if $a=0$ then $f(x)$ is not quadratic, so $c=1$ from $b=ab+bc$, we have $b=ab+b$, so $ab=0$, since $a\neq 0, b=0$

From $2c=a^2+b^2+c^2$, we have

$2=a^2+0+1$

So, $a^2=1$ and $a=\pm 1$

thus, $f(x)=x^2+1$ or $f(x)=-x^2+1$

Since, $f(-1)=0$, we have $f(x)=-x^2+1$

$\Rightarrow \quad f(x)=-x^2+1$
$\Rightarrow$ Range $=(-\infty, 1]$

Question22

If $\alpha \neq 0$ and zero are the roots of the equation $x^2-5kx+\left(6k^2-2k\right)=0$, then $\alpha=$

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Options:

A.

$\frac{1}{3}$

B.

1

C.

$\frac{5}{3}$

D.

5

Answer: C

Solution:

If $\alpha \neq 0$ and 0 are the roots of

$x^2 - 5kx + (6k^2 - 2k) = 0$, then find α.

Step 1: Use the fact that 0 is a root.

If $x = 0$ is a root, it should satisfy the equation:

$0^2 - 5k(0) + (6k^2 - 2k) = 0$

$\Rightarrow 6k^2 - 2k = 0$

$\Rightarrow 2k(3k - 1) = 0$

So $k = 0$ or $k = \frac{1}{3}$.

But if $k = 0$, the equation becomes $x^2 = 0$, giving a double root 0 — contradicting that one root is nonzero ($\alpha \neq 0$).

Hence,

$k = \frac{1}{3}$.

Step 2: Substitute $k = \frac{1}{3}$ into the equation.

$x^2 - 5\left(\frac{1}{3}\right)x + \left(6\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right)\right) = 0$

$\Rightarrow x^2 - \frac{5}{3}x + \left(\frac{6}{9} - \frac{2}{3}\right) = 0$

$\Rightarrow x^2 - \frac{5}{3}x + \left(\frac{2}{3} - \frac{2}{3}\right) = 0$

Wait — compute the constant term carefully:

$\frac{6}{9} - \frac{2}{3} = \frac{2}{3} - \frac{2}{3} = 0.$

So the equation becomes:

$x^2 - \frac{5}{3}x = 0$

$x(x - \frac{5}{3}) = 0$

Step 3: Identify the roots.

Roots are $x = 0$ and $x = \frac{5}{3}$.

Given that one root is 0 and another is $\alpha \neq 0$,

$\boxed{\alpha = \frac{5}{3}}$.

✅ **Final Answer:**

$\boxed{\alpha = \frac{5}{3}}$

Option C.

Question23

The set of all real values of x satisfying the inequation $\frac{8x^2-14x-9}{3x^2-7x-6} > 2$ is

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Options:

A.

$(-\infty, 1) \cup (3, \infty)$

B.

$\left(-\infty, -\frac{2}{3}\right) \cup (2, \infty)$

C.

$\left(-\frac{2}{3}, 2\right)$

D.

$\left(-\infty, -\frac{2}{3}\right) \cup (3, \infty)$

Answer: D

Solution:

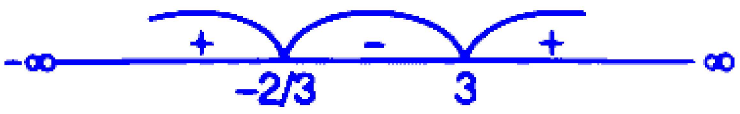
$$\frac{8x^2-14x-9}{3x^2-7x-6}-2>0$$

$$\Rightarrow \frac{8x^2-14x-9-6x^2+14x+12}{3x^2-7x-6}>0$$

$$\Rightarrow \frac{2x^2+3}{3x^2-9x+2x-6}>0$$

$$\Rightarrow \frac{2x^2+3}{3x(x-3)+2(x-3)}>0$$

$$\Rightarrow \frac{2x^2+3}{(x-3)(3x+2)}>0$$



$x \in \left(-\infty, \frac{-2}{3}\right) \cup (3, \infty)$

Question24

When the roots of $x^3+\alpha x^2+\beta x+6=0$ are increased by 1 , if one of the resultant values is the least root of $x^4-6x^3+11x^2-6x=0$, then

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Options:

A.

$\alpha-\beta+5=0$

B.

$\alpha+\beta+7=0$

C.

$2\alpha+\beta+7=0$

D.

$2\alpha+3\beta-1=0$

Answer: A

Solution:

$x\left(x^3-6x^2+11x-6\right)=0$

$$\begin{aligned} & \Rightarrow \quad x(x-1)(x-2)(x-3)=0 \\ & \Rightarrow \quad x=0,1,2,3\end{aligned}$$

The least roots is 0 .

Let the roots of $x^3+\alpha x^2+\beta x+6=0$ be r_1, r_2, r_3

The roots of $x^3+\alpha x^2+\beta x+6=0$

increased by 1 are r_1+1, r_2+1, r_3+1

One of these roots is 0 , so $r_i+1=0$. for some i^0.

$\Rightarrow r_i=-1$

Since $r_i=-1$ is root of

$$\begin{aligned} & x^3+\alpha x^2+\beta x+6=0 \\ & \Rightarrow(-1)^3+\alpha(-1)^2+\beta(-1)+6=0 \\ & \Rightarrow-1+\alpha-\beta+6=0 \\ & \Rightarrow \alpha-\beta+5=0\end{aligned}$$

Question25

Let ' a ' be a non-zero real number. If the equation whose roots are the squares of the roots of the cubic equation $x^3-a x^2+a x-1=0$ is identical with this cubic equation, then ' a ' =

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Options:

A.

$\frac{1}{3}$

B.

3

C.

$\frac{1}{2}$

D.

2

Answer: B

Solution:

$$
\begin{aligned}
&\alpha+\beta+\gamma=a \\
&\alpha\beta+\beta\gamma+\gamma\alpha=a \\
&\alpha\beta\gamma=1 \\
&\Rightarrow \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha) \\
&\quad \Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma) \\
&\qquad =a^2-2a \\
&\quad \therefore \alpha^2\beta^2\gamma^2=(\alpha\beta\gamma)^2=1^2=1
\end{aligned}
$$

Now, equate the coefficients of the original and new equations.

$$
\begin{aligned}
a^2-2a&=a \\
a^2-2a&=a \\
1&=1
\end{aligned}
$$

Solve for $\boldsymbol{a}$

$a^2-2a=a$

$$
\begin{aligned}
&\Rightarrow \quad a^2-3a=0 \\
&\Rightarrow \quad a(a-3)=0 \\
&\because \quad a\neq 0 \therefore a=3
\end{aligned}
$$

Question26

If $(2k-1)x^2-2(3k-2)x+4k>0$ for every $x\in R$, then the sum of all possible integral values of k is

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Options:

A.

21

B.

27

C.

36

D.

28

Answer: D

Solution:

We have,

$$(2k-1)x^2 - 2(3k-2)x + 4k > 0, \forall x \in \mathbb{R}$$

$$\Rightarrow \quad 4(3k-2)^2 - 4(2k-1)4k < 0$$

$$\Rightarrow \quad 9k^2 - 12k + 4 - 8k^2 + 4k < 0$$

$$\Rightarrow \quad k^2 - 8k + 4 < 0,$$

$$k = \frac{8 \pm \sqrt{64-16}}{2} = \frac{8 \pm 4\sqrt{3}}{2}$$

$$\Rightarrow \quad k \in (4 - 2\sqrt{3}, 4 + 2\sqrt{3})$$

$$k = 1, 2, 3, \ldots, 7$$

$$\therefore \quad \text{Sum of integral value } = 28$$

Question27

If α is a repeated root of multiplicity 2 of the equation $18x^3 - 33x^2 + 20x - 4 = 0$, then

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Options:

A.

$3\alpha^2 - 8\alpha + 4 = 0$

B.

$3\alpha^2 + 8\alpha + 4 = 0$

C.

$3\alpha^2 - \alpha - 4 = 0$

D.

$3\alpha^2 + 2\alpha - 4 = 0$

Answer: A

Solution:

$f(x)=18 x^3-33 x^2+20 x-4=0

α is a repeated root of multiplicity ? Let roots be α, α, β

$$f'(x)=54x^2-66x+20x=\alpha \text{ is repeated roots, } f'(\alpha)=0 \quad \begin{array}{ll} \text{and} & 54\alpha^2-66\alpha+20=0 \\ \Rightarrow & 27\alpha^2-33\alpha+10=0 \\ \Rightarrow & 27\alpha^2-18\alpha-15\alpha+10=0 \\ \Rightarrow & (9\alpha-5)(3\alpha-2)=0 \Rightarrow \alpha=\frac{2}{3} \text{ or } \frac{5}{9} \end{array}$$

So, $\alpha=\frac{2}{3} undefined \alpha^2-8\alpha+4=0$

Question28

The equation $6x^4-5x^3+13x^2-5x+6=0$ will have

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Options:

A.

only real roots

B.

only complex roots

C.

two real and two complex roots

D.

two real and two purely imaginary roots

Answer: B

Solution:

We start with the equation: $6x^4-5x^3+13x^2-5x+6=0$.

Divide both sides by x^2 (where $x \neq 0$) to get:

$6x^2 - 5x + 13 - \frac{5}{x} + \frac{6}{x^2} = 0$

Rewrite the expression by grouping terms:$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 13 = 0$

Let $t = x + \frac{1}{x}$. Then $x^2 + \frac{1}{x^2} = t^2 - 2$.

Substitute these into the equation:$6(t^2 - 2) - 5t + 13 = 0$

Simplify:$6t^2 - 12 - 5t + 13 = 06t^2 - 5t + 1 = 0$

This is a quadratic equation in t. The discriminant (D) is:

$D = (-5)^2 - 4 \cdot 6 \cdot 1 = 25 - 24 = 1$ (which is greater than 0).

This means there are two real solutions for t:$t = \frac{5 \pm 1}{12} = \frac{6}{12}$ or $\frac{4}{12}t = \frac{1}{2} \quad \text{or} \quad \frac{1}{3}$

So, $x + \frac{1}{x} = \frac{1}{2}$ or $x + \frac{1}{x} = \frac{1}{3}$.

Now solve for x in each case:

If $x + \frac{1}{x} = \frac{1}{2}$, multiply both sides by x: $x^2 + 1 = \frac{1}{2}x$. Rearranged:$2x^2 - x + 2 = 0$

If $x + \frac{1}{x} = \frac{1}{3}$, multiply both sides by x: $x^2 + 1 = \frac{1}{3}x$. Rearranged:$3x^2 - x + 3 = 0$

For both equations, the discriminant is less than 0. This means there are no real solutions for x.

Therefore, all the solutions for x are complex numbers.

Question29

The roots α, β of the equation $x^2 - 6(k-1)x + 4(k-2) = 0$ are equal in magnitude but opposite in sign, if $\alpha > \beta$, then the product of the roots of the equation $2x^2 - \alpha x + 6\beta(\alpha + 1) = 0$

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Options:

A.

12

B.

-12

C.

16

D.

-18

Answer: D

Solution:

Given, $x^2 - 6(k-1)x + 4(k-2) = 0$

$\alpha + \beta = 6(k-1) = 0 \Rightarrow k = 1$
$\alpha\beta = 4(k-2) < 0 \Rightarrow k < 2$

for $k = 1, x^2 - 4 = 0 \Rightarrow x^2 = 4$

$\therefore \quad x = \pm 2 \Rightarrow \alpha = +2$ and $\beta = -2$

Then, $2x^2 - \alpha x + 6\beta(\alpha + 1) = 0$ become

$$2x^2 - 2x + 6(-2)(3) = 0$$
$$x^2 - x - 18 = 0$$

Now, product of roots $= -18$

Question30

If $ax^2 + bx + c < 0 \forall x \in R$ and the expressions $cx^2 + ax + b$ and $ax^2 + bx + c$ have their extreme values at the same point x, then for the expression $cx^2 + ax + b$

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Options:

A.

Minimum value $= \frac{4b}{3}$

B.

Maximum value $= \frac{4a}{3}$

C.

Minimum value $= \frac{3a}{4}$

D.

Maximum value $= \frac{3b}{4}$

Answer: D

Solution:

We have, $ax^2 + bx + c < 0$

If $a < 0$, then $\Delta < 0$

$a < 0$ and $b^2 - 4ac < 0$

$\therefore b^2 < 4ac$

And $cx^2 + ax + b$ and $ax^2 + bx + c$ have same extreme value

$\because \quad -\frac{a}{2c} = -\frac{b}{2a} \Rightarrow a^2 = bc$

Now, the minimum value of $cx^2 + ax + b$ is $-\frac{\Delta}{4a} = -\frac{a^2-4bc}{4c} = -\frac{bc-4bc}{4c} = -\left(-\frac{3bc}{4c}\right) = \frac{3b}{4}$

Because $b^2 =$ positive and $a < 0 \Rightarrow c < 0$

$\therefore \frac{3b}{4}$ is the maximum value of $cx^2 + ax + b$

Question31

If $x^2 - 5x + 6$ is a factor of $f(x) = x^4 - 17x^3 + kx^2 - 247x + 210$, then the other quadratic factor of $f(x)$ is

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Options:

A.

$x^2 + 12x + 35$

B.

$x^2 - 12x + 35$

C.

$x^2 - 6x + 35$

D.

$x^2 + 6x + 35$

Answer: B

Solution:

$f(x) = x^4 - 17x^3 + kx^2 - 247x + 210$

Its factor: $x^2 - 5x + 6$

$\because (x-2)(x-3)$ is also a factor.

$$\begin{aligned} &\mathrm{f}(2) = 16 - 17(8) + k(4) - (247)2 + 210 = 0 \\ &\Rightarrow \quad 16 - 136 + 4k - 494 + 210 = 0 \\ &\Rightarrow \quad 4k - 404 = 0 \Rightarrow k = 101 \\ &\because f(x) = x^4 - 17x^3 + 101x^2 - 247x + 210 \\ &\quad = \left(x^2 - 5x + 6\right)\left(x^2 - 12x + 35\right) \end{aligned}$$

Question32

Given $f(x) = x^2 - 5x + 4$. Out of first 20 natural numbers, if a number x is chosen at random, then the probability that the chosen x satisfies the inequality $f(x) > 10$ is

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Options:

A.

$\frac{1}{2}$

B.

$\frac{3}{4}$

C.

$\frac{7}{10}$

D.

$\frac{13}{20}$

Answer: C

Solution:

Given, $f(x) = x^2 - 5x + 4$
$f(x) > 10$
$x^2 - 5x + 4 > 10$
$x^2 - 5x - 6 > 0$

$(x - 6)(x + 1) > 0$
$x < -1$ or $x > 6$
$n(S) = {}^{20}C_1 = 20$
$n(E) = {}^{14}C_1 = 14$
$P(E) = \frac{14}{20} = \frac{7}{10}$

Question33

If the harmonic mean of the roots of the equation $\sqrt{2}x^2 - bx + (8 - 2\sqrt{5}) = 0$ is

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Options:

A.

3

B.

2

C.

$4 - \sqrt{5}$

D.

$4 + \sqrt{5}$

Answer: C

Solution:

$\sqrt{2}x^2 - bx + (8 - 2\sqrt{5}) = 0$

Let it's root's be α and β,

$\alpha + \beta = \frac{b}{\sqrt{2}}, \quad \alpha\beta = \frac{8-2\sqrt{5}}{\sqrt{2}}$

The harmonic mean of the roots,

$$\begin{aligned} H &= \frac{2\alpha\beta}{\alpha+\beta} = 4 \\ \alpha\beta &= 2(\alpha+\beta) \\ \frac{8-2\sqrt{5}}{\sqrt{2}} &= 2 \cdot \left(\frac{b}{\sqrt{2}}\right) \\ 2b &= 2(4-\sqrt{5}) \\ b &= 4-\sqrt{5} \end{aligned}$$

Question34

All the values of k such that the quadratic expression $2kx^2 - (4k+1)x + 2$ is negative for exactly three integrals values of x, lie in the interval

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Options:

A.

$\left[\frac{1}{12}, \frac{1}{10}\right)$

B.

$\left(\frac{1}{6}, \frac{1}{5}\right)$

C.

$[-1, 2)$

D.

$[2, 6)$

Answer: A

Solution:

We have,

$$\begin{aligned} 2kx^2 - (4k+1)x + 2 < 0 \\ \because 2kx^2 - 4kx - x + 2 < 0 \\ 2kx(x-2) - 1(x-2) < 0 \\ (x-2)(2kx-1) < 0 \\ 2 < x < \frac{1}{2k} \end{aligned}$$

Expression is negative for exactly 3 integral values of x which is $x = 3, 4, 5$

$$\begin{aligned} & 5 < x \leq 6 \\ \because \quad & \tfrac{1}{2k} \leq 6 \Rightarrow k \geq \tfrac{1}{12} \\ \therefore \quad & \tfrac{1}{2k} > 5 \Rightarrow k < \tfrac{1}{10} \\ \therefore \quad & k \in \left[\tfrac{1}{12}, \tfrac{1}{10}\right) \end{aligned}$$

Question35

If α, β and γ are the roots of the equation $x^3 - 13x^2 + kx + 189 = 0$ such that $\beta - \gamma = 2$, then $\beta + \gamma : k + \alpha =$

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Options:

A.

4 : 3

B.

2 : 1

C.

6 : 5

D.

3 : 4

Answer: A

Solution:

Given, $x^3-13x^2+kx+189=0$

$\alpha+\beta+\gamma=13, \alpha\beta+\beta\gamma+\gamma\alpha=k$
and $\quad \alpha\beta\gamma=-189$ and $\beta-\gamma=2$
$[\because \gamma=\beta-2]$
$\alpha+\beta+\beta-2=13$
$\Rightarrow \quad (15-2\beta)\beta(\beta-2)=-189$
$\Rightarrow \quad \beta\left(15\beta-2\beta^2-30+4\beta\right)=-189$
$\Rightarrow \quad -2\beta^3+19\beta^2-30\beta+189=0$
$\Rightarrow \quad 2\beta^3-19\beta^2+30\beta-189=0$
$\Rightarrow \quad (\beta-9)\left(2\beta^2-\beta+21\right)=0$
$\Rightarrow \quad \beta=9, 2\beta^2-\beta+21\neq 0 \forall \beta\in R$
$\therefore \quad \beta=9, \gamma=7$

and $\quad \alpha=-3$

$k=\alpha\beta+\beta\gamma+\gamma\alpha=-27+63-21=15$

Now, $\frac{\beta+\gamma}{k+\alpha}=\frac{9+7}{15-3}=\frac{16}{12}=\frac{4}{3}$

$\therefore \quad \beta+\gamma : k+\alpha=4:3$

Question36

The set of all real values of x satisfying the inequality $\frac{7x^2-5x-18}{2x^2+x-6}<2$ is

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Options:

A. $\left(-\infty,-\frac{2}{3}\right] \cup [3, \infty)$

B. $\left(-2,-\frac{2}{3}\right) \cup \left(\frac{3}{2}, 3\right)$

C. $(-\infty,-2) \cup \left(\frac{3}{2}, \infty\right)$

D. $\left[-\frac{2}{3}, \frac{3}{2}\right)$

Answer: B

Solution:

We have, $\frac{7x^2-5x-18}{2x^2+x-6}<2$

$$\frac{7x^2-5x-18}{2x^2+x-6}-2<0$$
$$\frac{7x^2-5x-18-4x^2-2x+12}{2x^2+x-6}<0$$
$$\frac{3x^2-7x-6}{2x^2+x-6}<0$$
$$\frac{3x^2-9x+2x-6}{2x^2+4x-3x-6}<0$$
$$\frac{(3x+2)(x-3)}{(2x-3)(x+2)}<0$$

Case I

$(3x+2)(x-3)<0$ and
$(2x-3)(x+2)>0$
$$\Rightarrow x \in \left(\frac{-2}{3}, 3\right) \text{ and } x \in (-\infty,-2) \cup \left(\frac{3}{2}, 0\right)$$
$$\Rightarrow x \in \left(\frac{3}{2}, 3\right)$$

Case II

$(3x+2)(x-3)>0$
and $(2x-3)(x+2)<0$
$$\Rightarrow \quad x \in \left(-\infty,-\frac{2}{3}\right) \cup (3, \infty) \text{ and}$$
$$\Rightarrow \quad x \in \left(-2, \frac{3}{2}\right)$$
$$\Rightarrow \quad x \in \left(-2,-\frac{2}{3}\right)$$

Hence, $x \in \left(-2,-\frac{2}{3}\right) \cup\left(\frac{3}{2}, 3\right)$

Question37

The set of all values of k for which the inequality $x^2-(3k+1)x+4k^2+3k-3>0$ is true for all real values of x, is

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Options:

A. $\left(-\frac{13}{7}, 1\right)$

B. $\left(-1, \frac{13}{7}\right)$

C. $\left(-\infty,-\frac{13}{7}\right) \cup(1, \infty)$

D. $(-\infty, -1) \cup \left(\frac{13}{7}, \infty\right)$

Answer: C

Solution:

To determine the set of all values of k for which the inequality $x^2 - (3k+1)x + 4k^2 + 3k - 3 > 0$ holds for all real values of x, we proceed as follows.

First, recall that the inequality $f(x) > 0$ for all x requires that the quadratic equation has no real roots, which corresponds to the discriminant D being less than zero. For a quadratic equation $ax^2 + bx + c$, the discriminant is given by $b^2 - 4ac$.

In this scenario, the quadratic in x is given by:

$$a = 1, \quad b = -(3k+1), \quad c = 4k^2 + 3k - 3.$$

The discriminant D is:

$$D = (3k+1)^2 - 4(1)(4k^2 + 3k - 3).$$

Simplifying, we get:

$$D = (3k+1)^2 - 16k^2 - 12k + 12.$$

Expanding:

$$D = 9k^2 + 6k + 1 - 16k^2 - 12k + 12.$$

Simplifying further:

$$D = -7k^2 - 6k + 13.$$

To ensure $D < 0$, we need:

$$7k^2 + 6k - 13 > 0.$$

To solve this inequality, factor the quadratic expression:

$$7k^2 + 13k - 7k - 13 = (7k+13)(k-1).$$

The solution to $(7k+13)(k-1) > 0$ can be found using sign analysis. The critical points are $k = -\frac{13}{7}$ and $k = 1$.

The intervals to consider are:

$k < -\frac{13}{7}$

$-\frac{13}{7} < k < 1$

$k > 1$

Test each interval:

For $k < -\frac{13}{7}$ and $k > 1$, both factors $(7k+13)$ and $(k-1)$ have the same sign (either both positive or both negative), making the product positive.

For $-\frac{13}{7} < k < 1$, the factors have opposite signs, making the product negative.

Thus, the solution set for k is:

$k \in \left(-\infty, -\frac{13}{7}\right) \cup (1, \infty)$.

Question38

The cubic equation whose roots are the square of the roots of the equation is

$$12x^3 - 20x^2 + x + 3 = 0$$

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Options:

A. $x^3 + 376x^2 - 121x - 9 = 0$

B. $144x^3 - 400x^2 + 121x + 98 = 0$

C. $144x^3 - 376x^2 + 121x - 9 = 0$

D. $x^3 + 400x^2 - 121x - 98 = 0$

Answer: C

Solution:

To find the cubic equation whose roots are the squares of the roots of the given equation, we start by analyzing the equation:

$12x^3 - 20x^2 + x + 3 = 0$

Let's consider α, β, and γ as the roots of this equation. From the properties of polynomial roots, we have:

$\alpha + \beta + \gamma = \frac{20}{12} = \frac{5}{3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{12}$

$\alpha\beta\gamma = -\frac{3}{12} = -\frac{1}{4}$

Next, we need to find relationships involving the squares of these roots α^2, β^2, and γ^2.

Sum of squares of the roots:

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= \left(\frac{5}{3}\right)^2 - 2 \times \frac{1}{12} = \frac{25}{9} - \frac{1}{6} = \frac{50-3}{18} = \frac{47}{18}$

Product of squares of the roots:

$\alpha^2\beta^2\gamma^2 = (\alpha\beta\gamma)^2 = \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$

Sum of products of squares of roots taken two at a time:

$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$= \left(\frac{1}{12}\right)^2 - 2 \times \left(-\frac{1}{4}\right) \times \frac{5}{3}$

$= \frac{1}{144} + \frac{5}{6} = \frac{1+120}{144} = \frac{121}{144}$

Using these calculated values, the cubic equation whose roots are the squares of the original roots is given by:

$x^3 - \frac{47}{18}x^2 + \frac{121}{144}x - \frac{1}{16} = 0$

Multiplying through by 144 to clear fractions, we obtain:

$144x^3 - 376x^2 + 121x - 9 = 0$

Question39

α, β and γ are the roots of the equation $x^3 + 3x^2 - 10x - 24 = 0$ If $\alpha(\beta + \gamma), \beta(\gamma + \alpha)$ and $\gamma(\alpha + \beta)$ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then q is equal to

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Options:

A. -44

B. -28

C. 44

D. 28

Answer: D

Solution:

Let α, β and γ be the roots of equation $x^3 + 3x^2 - 10x - 24 = 0$

$\alpha + \beta + \gamma = -3, \alpha\beta + \beta\gamma + \gamma\alpha = -10$

and $\alpha\beta\gamma = 24$

$\Rightarrow \quad \alpha = -2, \beta = -4, \gamma = 3$

As, $\alpha(\beta+\gamma), \beta(\gamma+\infty)$ and $\gamma(\alpha+\beta)$ are the roots of equation

$$\begin{aligned} & x^3+px^2+qx+r=0 \\ & -p=\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta) \\ & \quad=-2(-4+3)-4(-2+3)+3(-2-4) \\ & \quad=2-4-18 \Rightarrow -20 \\ \therefore \quad & p=20 \\ & q=2(-4)+(-4)(-18)+(-18)2 \\ & \quad=-8+72-36=28 \end{aligned}$$

Question40

If ' a ' is a rational number, then the roots of the equation $x^2-3ax+a^2-2a-4=0$ are

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Options:

A. rational and equal numbers

B. different real numbers

C. different rational numbers only

D. not real numbers

Answer: B

Solution:

Given,

$x^2-3ax+a^2-2a-4=0$

where, a is a rational number

Then,

discriminant, $D=B^2-4AC$

$$\begin{aligned} & =(-3a)^2-4\times 1\times\left(a^2-2a-4\right) \\ & =9a^2-4a^2+8a+16 \\ & =5a^2+8a+16 \\ D & =\underbrace{5a^2+16+8a}_{+\mathrm{ve}} \end{aligned}$$

So, $D>0$

So, roots of equation will be different real number.

Question41

The set of all real values ' a ' for which $-1<\frac{2x^2+ax+2}{x^2+x+1}<3$ holds for all real values of x is

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Options:

A. $(-7,5)$

B. $(5,\infty)$

C. $(1,5)$

D. $(-\infty,1)$

Answer: C

Solution:

$$\text{Given, } -1<\frac{2x^2+ax+2}{x^2+x+1}<3$$

$$\text{Now, } -1<\frac{2x^2+ax+2}{x^2+x+1}$$

$$\begin{aligned}
&\Rightarrow \quad -x^2-x-1<2x^2+ax+2\\
&\Rightarrow \quad 0<3x^2+(a+1)x+3\\
&\Rightarrow \quad D<0\\
&\Rightarrow \quad (a+1)^2-4\times3\times3<0\\
&\Rightarrow \quad (a+1)^2-36<0\Rightarrow(a+1)^2-6^2<0\\
&\Rightarrow \quad (a+1-6)(a+1+6)<0\\
&\Rightarrow \quad (a-5)(a+7)<0
\end{aligned}$$

$$\text{and } \frac{2x^2+ax+2}{x^2+x+1}<3 \qquad \ldots(i)$$

$$\begin{aligned}
&\Rightarrow \quad 2x^2+ax+2<3x^2+3x+3\\
&\Rightarrow \quad 0<x^2+(3-a)x+1\Rightarrow D<0\\
&\Rightarrow \quad (3-a)^2-4\times1\times1<0\\
&\Rightarrow \quad (3-a)^2-2^2<0\\
&\Rightarrow \quad (3-a-2)(3-a+2)<0\\
&\Rightarrow \quad (-a+1)(5-a)<0
\end{aligned}$$

$\Rightarrow \quad (a-5)(-1+a)<0 \qquad \ldots(ii)$

From Eq. (i) and (ii), we get

$a \in (1,5)$

Question42

The quotient, when $3x^5 - 4x^4 + 5x^3 - 3x^2 + 6x - 8$ is divided by $x^2 + x - 3$ is

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Options:

A. $3x^2 - 7x - 21$

B. $3x^3 - 7x^2 + 21x - 45$

C. $3x^4 - 7x^3 + 21x^2 - 45 + 114$

D. $114x - 143$

Answer: B

Solution:

$x^2 + x - 3)$

$3x^5 - 4x^4 + 5x^3 - 3x^2 + 6x - 8$

$(3x^3 - 7x^2 + 21x - 45$

$3x^5+3x^4-9x^3$

$-\quad -\quad +$

$-7x^4+14x^3-3x^2+6x-8$

$-7x^4-7x^3+21x^2$

$+\quad +\quad -$

$21x^3-24x^2+6x-8$

$21x^3-21x^2+63x$

$-\quad +\quad +$

$-45x^2+69x-8$

$-45x^2+45x-135$

$+\quad +\quad +$

$114x+127$

Hence, quotient is $3x^3-7x^2+21x-45$

Question43

If both the roots of the equation $x^2-6ax+2-2a+9a^2=0$ exceed 3 , then

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Options:

A. $a<\frac{3}{2}$

B. $a>\frac{3}{2}$

C. $a<\frac{5}{2}$

D. $a>\frac{11}{9}$

Answer: D

Solution:

Given, $x^2-6ax+2-2a+9a^2=0$

$\Rightarrow \quad x^2-6ax+\left(2-2a+9a^2\right)=0$

For both roots to exceed 3, $x = 3$ should be a root or both roots should be greater than 3.

Sum of roots $= -\frac{b}{a} = -\left(\frac{-6a}{1}\right) = 6a$

$6a > 18 \Rightarrow a > 3$

Product of roots $= \frac{c}{a} = \frac{9a^2-2a+2}{1}$

Since, $a > 3$ so product, also should be greater than 3 .

That leads us to the correct condition

$a > \frac{11}{9}$

Question44

If α and β are two distinct negative roots of $x^5 - 5x^3 + 5x^2 - 1 = 0$, then the equation of least degree with integer coefficients having $\sqrt{-\alpha}$ and $\sqrt{-\beta}$ as its roots, is

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Options:

A. $x^2 - 3x + 1 = 0$

B. $-x^4 - 5x^2 + 5x + 1 = 0$

C. $-x^4 + 5x^2 - 5x + 1 = 0$

D. $x^4 - 3x^2 + 1 = 0$

Answer: D

Solution:

We begin with the polynomial:

$x^5 - 5x^3 + 5x^2 - 1 = (x-1)^3(x^2 + 3x + 1)$

We are given that α and β are distinct negative roots of the equation. Therefore, α and β must be the roots of the quadratic equation:

$x^2 + 3x + 1 = 0$

Given that α and β are negative, we establish that:

$\alpha\beta = 1 \quad$ (as derived from the product of the roots of $x^2 + 3x + 1 = 0$)

We need to find the polynomial equation for which $\sqrt{-\alpha}$ and $\sqrt{-\beta}$ are roots. Note:

$\sqrt{-\alpha} \cdot \sqrt{-\beta} = \sqrt{\alpha \cdot \beta} = \sqrt{1} = 1$

Thus, the product of $\sqrt{-\alpha}$ and $\sqrt{-\beta}$ is 1. The equation with these roots and integer coefficients is:

$x^4 - 3x^2 + 1 = 0$

The product of the roots for this equation equals 1, satisfying the condition $\sqrt{-\alpha} \cdot \sqrt{-\beta} = 1$, confirming that $\sqrt{-\alpha}$ and $\sqrt{-\beta}$ are roots of the equation.

Question45

If α is a common root of $x^2 - 5x + \lambda = 0$ and $x^2 - 8x - 2\lambda = 0 (\lambda \neq 0)$ and β, γ are the other roots of them, then $\alpha + \beta + \gamma + \lambda =$

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Options:

A. 0

B. -1

C. 1

D. 2

Answer: C

Solution:

Given,

$$\begin{aligned} &x^2 - 5x + \lambda = 0 \qquad \ldots(i) \\ &x^2 - 8x - 2\lambda = 0 \qquad \ldots(ii) \end{aligned}$$

$\because$ α is a common factor of Eqs (i) and (ii), we get

And β, γ are the other roots of them,

So

$$\begin{array}{l} \alpha^2 - 5\alpha + \lambda = 0 \\ \alpha^2 - 8\alpha - 2\lambda = 0 \\ \quad -(+) \quad (+) \\ \hline \qquad 3\alpha + 3\lambda = 0 \Rightarrow \lambda = -\alpha \end{array}$$

Then, $\alpha^2 - 5\alpha - \alpha = 0 \Rightarrow \alpha^2 - 6\alpha = 0$

$$\Rightarrow \quad \alpha(\alpha-6)=0 \Rightarrow \alpha=0,6$$
$$\alpha \neq 0$$
$$\Rightarrow \alpha=6 \text{ and } \lambda=-6$$

Now, from Eq. (i), we get

$$x^2-5x-6=0$$
$$\Rightarrow \quad x^2-6x+x-6=0$$
$$\Rightarrow \quad (x-6)(x+1)=0$$
$$\Rightarrow \quad x=6, x=-1$$

$$\therefore \quad \beta=-1$$

From Eq. (ii), we get

$$x^2-8x-2\times(-6) \quad =0$$
$$\Rightarrow \quad x^2-8x+12 \quad =0$$
$$\Rightarrow \quad (x-6)(x-2) \quad =0 \Rightarrow x=6,2$$

$$\therefore \quad \gamma=2$$

$$Now, \quad \alpha+\beta+\gamma+\lambda=6+(-1)+2+(-\theta)$$
$$=1$$

Question46

The equation $x^4-x^3-6x^2+4x+8=0$ has two equal roots. If α, β are the other two roots of this equation, then $\alpha^2+\beta^2=$

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Options:

A. 4

B. 5

C. 6

D. 7

Answer: B

Solution:

To solve the given polynomial equation $x^4-x^3-6x^2+4x+8=0$ with two equal roots, we proceed as follows:

The equation can be expressed as:

$x^3(x+1) - 2x^2(x+1) - 4x(x+1) + 8(x+1) = 0$

Factoring out $(x+1)$ gives:

$(x+1)(x^3 - 2x^2 - 4x + 8) = 0$

Further factoring results in:

$(x+1)(x-2)(x^2-4) = 0$

This simplifies to:

$x+1 = 0, \quad x-2 = 0, \quad x^2 - 4 = 0$

Hence, the roots are:

$x = -1, \quad x = 2, \quad x = \pm 2$

The distinct roots of the equation are -2, -1, and 2. Since one root, 2, repeats, we determine the remaining roots as $\alpha = -2$ and $\beta = -1$.

To find $\alpha^2 + \beta^2$, compute:

$\alpha^2 + \beta^2 = (-2)^2 + (-1)^2 = 4 + 1 = 5$

Thus, $\alpha^2 + \beta^2 = 5$.

Question47

Roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are

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Options:

A. $\frac{a(b-c)}{c(a-b)}, 1$

B. $\frac{b(c-a)}{c(a-b)}, 1$

C. $\frac{c(a-b)}{a(b-c)}, 1$

D. $\frac{c(a-b)}{b(c-a)}, 1$

Answer: C

Solution:

We have,

$a(b-c)x^2+b(c-a)x+c(a-b)=0$

Using hit and trial method,

Put $x=1$ in Eq. (i)

$$a(b-c)+b(c-a)+c(a-b)=0$$
$$ab-ac+bc-ab+ac-bc=0$$

$\therefore$ one root is 1 .

Now, let another root be α

Then, product of root, $\alpha \cdot 1=\frac{c(a-b)}{a(b-c)}$

$\Rightarrow \quad \alpha=\frac{c(a-b)}{a(b-c)}$

Hence, $\frac{c(a+b)}{a(b-c)}, 1$

Question48

The algebraic equation of degree 4 whose roots are translate of the roots of the equation. $x^4+5x^3+6x^2+7x+9=0$ by -1 is

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Options:

A. $x^4+x^3-3x^2+6x+4=0$

B. $x^4+9x^3+27x^2+38x+28=0$

C. $x^4+5x^3+6x^2+7x+9=0$

D. $x^4-5x^3+6x^2-7x+9=0$

Answer: A

Solution:

To find the algebraic equation of degree 4 whose roots are translations of the roots of the given equation $x^4+5x^3+6x^2+7x+9=0$ by -1, we perform a substitution where x becomes $x-1$.

Given the equation:

$x^4+5x^3+6x^2+7x+9=0,$

we translate the roots by -1, meaning we consider $x=t-1$, which implies $t=x+1$.

We need to find $f(t-1)=0$:

$$\begin{aligned} f(t-1) &= (t-1)^4 + 5(t-1)^3 + 6(t-1)^2 + 7(t-1) + 9 \\ &= (t^4 - 4t^3 + 6t^2 - 4t + 1) \\ &\quad + 5(t^3 - 3t^2 + 3t - 1) \\ &\quad + 6(t^2 - 2t + 1) \\ &\quad + 7(t-1) + 9. \end{aligned}$$

Simplifying each expression:

$(t^4 - 4t^3 + 6t^2 - 4t + 1)$

$5(t^3 - 3t^2 + 3t - 1) = 5t^3 - 15t^2 + 15t - 5$

$6(t^2 - 2t + 1) = 6t^2 - 12t + 6$

$7(t-1) = 7t - 7$

Adding these terms together:

$$\begin{aligned} & t^4 - 4t^3 + 6t^2 - 4t + 1 \\ & + 5t^3 - 15t^2 + 15t - 5 \\ & + 6t^2 - 12t + 6 \\ & + 7t - 7 + 9 = 0 \end{aligned}$$

Combine like terms:

$$\begin{aligned} & t^4 + (5t^3 - 4t^3) + (6t^2 - 15t^2 + 6t^2) + (-4t + 15t - 12t + 7t) + (1 - 5 + 6 - 7 + 9) \\ & = t^4 + t^3 - 3t^2 + 6t + 4 = 0. \end{aligned}$$

Thus, the required equation is:

$x^4 + x^3 - 3x^2 + 6x + 4 = 0.$

Question49

Let $[r]$ denote the largest integer not exceeditio r and the roots of the equation $3x^2 + 6x + 5 + \alpha\left(x^2 + 2x + 2\right) = 0$ are complex number when ever $\alpha > L$ and $\alpha

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Options:

A. L

B. M

C. $L + M$

D. $M - L$

Answer: A

Solution:

Given equation is

$$3x^2+6x+5+\alpha\left(x^2+2x+2\right)=0$$
$$\Rightarrow(3+\alpha)x^2+(6+2\alpha)x+(5+2\alpha)=0$$

Now, discriminants,

$$b^2-4ac=(6+2\alpha)^2-4(3+\alpha)(5+2\alpha)$$
$$=36+4\alpha^2+24\alpha-60-44\alpha-8\alpha^2$$
$$=-4\alpha^2-20\alpha-24$$

Since, root are complex, so, discriminant is negative

$$-4\alpha^2-20\alpha-24<0$$
$$\Rightarrow\quad \alpha^2+5\alpha+6>0$$
$$\Rightarrow\quad (\alpha+2)(\alpha+3)>0$$
$$\Rightarrow\quad \alpha<-3 \text{ and } \alpha>-2$$

On comparing, we get

$M=-3, L=-2$

Now, $L-M=-2-(-3)=-2+3=1$

Now, $Ly^2+My+r<0$

$\Rightarrow\quad -2y^2-3y+r<0$

$\Rightarrow\quad 2y^2+3y-r>0$

So, $9+8r<0=0\Rightarrow r<-9/8$

Hence, $[r]=-2=L$

Question50

For any real value of x. If $\frac{11x^2+12x+6}{x^2+4x+2}\notin(a,b)$, then the value x for which $\frac{11x^2+12x+6}{x^2+4x+2}=b-a+3$ is

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Options:

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. 2

D. $-\frac{1}{2}$

Answer: D

Solution:

To solve the given problem, we start by defining the expression:

$y = \frac{11x^2+12x+6}{x^2+4x+2}$

This can be rewritten as:

$y(x^2 + 4x + 2) = 11x^2 + 12x + 6$

Rearranging the equation, we get:

$(y - 11)x^2 + (4y - 12)x + (2y - 6) = 0$

We will analyze this by considering two cases:

Case I: $y \neq 11$

For this quadratic equation in x, the discriminant D must be non-negative for real solutions:

$D = (4y - 12)^2 - 4(y - 11)(2y - 6) \geq 0$

Expanding this, we get:

$$\begin{aligned} D &= (4y - 12)^2 - 4(y - 11)(2y - 6) \\ &= 16y^2 - 96y + 144 - 8y^2 + 44y - 264 \\ &= 8y^2 - 52y - 120. \end{aligned}$$

Simplifying, we have:

$y^2 + 2y - 15 \geq 0.$

Factorizing, we find:

$(y + 5)(y - 3) \geq 0.$

This inequality implies:

$y \leq -5 \quad \text{or} \quad y \geq 3.$

Case II: $y = 11$

If $y = 11$, the quadratic equation becomes linear, and from the simplified version:

$(44y - 12)x + 22y - 6 = 0$

Simplifying gives:

$(32)x + 16 = 0$

Solving for x, we get:

$x = -\frac{1}{2}$.

Substituting $x = -\frac{1}{2}$ back into the expression for y:

$y = \frac{11\left(-\frac{1}{2}\right)^2 + 12\left(-\frac{1}{2}\right) + 6}{\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 2} = 11 \notin (-5, 3)$.

Thus, the value of x for which $y = b - a + 3$ is:

$x = -\frac{1}{2}$.

Question51

If the roots of $\sqrt{\frac{1-y}{y}} + \sqrt{\frac{y}{1-y}} = \frac{5}{2}$ are α and $\beta(\beta > \alpha)$ and the equation $(\alpha + \beta)x^4 - 25\alpha\beta x^2 + (\gamma + \beta - \alpha) = 0$ has real roots, then a possible value of γ is

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Options:

A. $\frac{1}{2}$

B. 4

C. 2π

D. $\sqrt{e + 13}$

Answer: A

Solution:

To solve the problem, we begin by analyzing the equation:

$\frac{\sqrt{1-y}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{1-y}} = \frac{5}{2}$

This simplifies to:

$\frac{1-y+y}{\sqrt{y(1-y)}} = \frac{5}{2}$

Hence,

$\frac{1}{\sqrt{y(1-y)}} = \frac{5}{2}$

Squaring both sides gives:

$4 = 25y(1 - y)$

Simplifying, we obtain:

$25y^2 - 25y + 4 = 0$

Solving this quadratic equation yields:

$y = \frac{4}{5}, \frac{1}{5}$

Thus, the roots are $\alpha = \frac{1}{5}$ and $\beta = \frac{4}{5}$ with $\beta > \alpha$.

Next, consider the given equation:

$(\alpha + \beta)x^4 - 25\alpha\beta x^2 + (\gamma + \beta - \alpha) = 0$

Substituting $\alpha = \frac{1}{5}$ and $\beta = \frac{4}{5}$, we have:

$x^4 - 4x^2 + \left(\gamma + \frac{3}{5}\right) = 0$

For this equation to have real roots, the discriminant D of the quadratic x^2 term must be non-negative:

$D = 16 - 4\left(\gamma + \frac{3}{5}\right) \geq 0$

Solving for γ, we find:

$4\left(\gamma + \frac{3}{5}\right) \leq 16$

$\gamma \leq 4 - \frac{3}{5} = \frac{17}{5} = 3.4$

Thus, a possible value of γ is $\frac{1}{2}$.

Question52

If α and β are two double roots of $x^2 + 3(a+3)x - 9a = 0$ for different values of $a (\alpha > \beta)$, then the minimum value of $x^2 + \alpha x - \beta = 0$ is

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Options:

A. $\frac{69}{4}$

B. $-\frac{69}{4}$

C. $-\frac{35}{4}$

D. $\frac{35}{4}$

Answer: B

Solution:

To solve the problem, we start with the given quadratic equation where α and β are double roots for different values of a:

$x^2 + 3(a+3)x - 9a = 0.$

For the roots to be double, the discriminant D must equal zero:

$D = (3(a+3))^2 - 4 \times 1 \times (-9a) = 0.$

This simplifies to:

$9(a+3)^2 + 36a = 0.$

Expanding and simplifying, we get:

$9(a^2 + 6a + 9) + 36a = 0,$

$9a^2 + 54a + 81 + 36a = 0,$

$9a^2 + 90a + 81 = 0.$

Divide the entire equation by 9:

$a^2 + 10a + 9 = 0.$

Solving this quadratic equation:

$a = \frac{-10 \pm \sqrt{100-36}}{2} = \frac{-10 \pm \sqrt{64}}{2}.$

$a = \frac{-10 \pm 8}{2}.$

This results in $a = -1$ and $a = -9$.

For $a = -9$:

The quadratic becomes:

$x^2 + 3(-9+3)x + 81 = 0 \quad \Rightarrow \quad x^2 - 18x + 81 = 0.$

The double root is:

$x = 9.$

For $a = -1$:

The quadratic becomes:

$x^2 + 3(2)x + 9 = 0 \quad \Rightarrow \quad x^2 + 6x + 9 = 0.$

The double root is:

$x = -3.$

From this, we identify $\alpha = 9$ and $\beta = -3$.

Now consider the equation:

$x^2 + \alpha x - \beta = 0 \quad \Rightarrow \quad x^2 + 9x + 3 = 0.$

The minimum value of a quadratic equation $ax^2 + bx + c$ is given by:

$c - \frac{b^2}{4a}.$

Plugging in the coefficients:

$= 3 - \frac{81}{4} = -\frac{69}{4}.$

Question53

If $2x^2 + 3x - 2 = 0$ and $3x^2 + \alpha x - 2 = 0$ have one common root, then the sum of all possible values of α is

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Options:

A. -35

B. 7.5

C. -7.5

D. -1.5

Answer: B

Solution:

To solve the problem, we have two quadratic equations, one of which is:

$2x^2 + 3x - 2 = 0$

First, we find the roots of this equation using the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substituting the specific values:

$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-2)}}{2 \times 2}$

$x = \frac{-3 \pm \sqrt{9+16}}{4}$

$x = \frac{-3 \pm \sqrt{25}}{4}$

$x = \frac{-3 \pm 5}{4}$

Thus, the roots of the first equation are:

$x = \frac{2}{4}, \frac{-8}{4} = \frac{1}{2}, -2$

Given that the second equation:

$3x^2 + \alpha x - 2 = 0$

shares one root with the first equation, we need to substitute these roots into the second equation.

Case 1: Root $x = \frac{1}{2}$

Substitute into the second equation:

$3\left(\frac{1}{2}\right)^2 + \alpha\left(\frac{1}{2}\right) - 2 = 0$

$3 \times \frac{1}{4} + \frac{1}{2}\alpha - 2 = 0$

$\frac{3}{4} + \frac{\alpha}{2} - 2 = 0$

Solving for α:

$\frac{\alpha}{2} = 2 - \frac{3}{4}$

$\frac{\alpha}{2} = \frac{5}{4}$

$\alpha = \frac{5}{2}$

Case 2: Root $x = -2$

Substitute into the second equation:

$3(-2)^2 + \alpha(-2) - 2 = 0$

$12 - 2\alpha - 2 = 0$

$2\alpha = 10$

$\alpha = 5$

Sum of All Possible Values of α

Adding the solutions:

$\alpha = \frac{5}{2} + 5 = \frac{5}{2} + \frac{10}{2} = \frac{15}{2} = 7.5$

Thus, the sum of all possible values of α is 7.5.

Question54

If the sum of two roots of $x^3 + px^2 + qx - 5 = 0$ is equal to its third root, then $p\left(p^2 - 4q\right) =$

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Options:

A. -20

B. 20

C. 40

D. -40

Answer: C

Solution:

Let α, β, γ be the roots of the equation $x^3 + px^2 + qx - 5 = 0$.

Given that $\alpha + \beta = \gamma$, we can derive the following:

From Vieta's formulas, we know:

$\alpha + \beta + \gamma = -p$

Substituting the given condition $\alpha + \beta = \gamma$ into the equation, we have:

$$\begin{aligned} \alpha + \beta + \gamma = \gamma + \gamma = 2\gamma, \\ 2\gamma = -p, \\ \gamma = \frac{-p}{2}. \end{aligned}$$

Another Vieta formula gives us:

$\alpha\beta + \beta\gamma + \gamma\alpha = q.$

Substituting for $\alpha + \beta = \gamma$, we get:

$$\begin{aligned} \alpha\beta + \gamma(\beta + \alpha) = q, \\ \alpha\beta + \gamma^2 = q, \\ \alpha\beta + \frac{p^2}{4} = q. \end{aligned}$$

Thus, $\alpha\beta = q - \frac{p^2}{4}$.

Using the product of roots from Vieta's formula:

$\alpha\beta\gamma = 5.$

Substituting $\gamma = \frac{-p}{2}$ and $\alpha\beta = q - \frac{p^2}{4}$, we have:

$$\left(q-\frac{p^2}{4}\right)\left(-\frac{p}{2}\right)=5,$$
$$\left(\frac{4q-p^2}{4}\right)\left(-\frac{p}{2}\right)=5,$$
$$\frac{-p(4q-p^2)}{8}=5,$$
$$p(4q-p^2)=-40,$$
$$p(p^2-4q)=40.$$

Thus, the value is 40.

Question55

$$4+\frac{1}{4+\frac{1}{4+\frac{1}{4+\ldots\infty}}}=$$

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Options:

A. $(2+\sqrt{5}),(2-\sqrt{5})$

B. $2+\sqrt{5}$

C. $2-\sqrt{5}$

D. $2+\sqrt{3}$

Answer: B

Solution:

Let $x=4+\frac{1}{4+\frac{1}{4+\frac{1}{4+\frac{1}{4+\ldots\infty}}}}$

$$\Rightarrow \quad x=4+\frac{1}{x}$$
$$\Rightarrow \quad x^2-4x-1=0$$
$$\Rightarrow \quad x=\frac{4\pm\sqrt{16+4}}{2}=2\pm\sqrt{5}$$

As the value of x is greater than 4, $x=2+\sqrt{5}$

Question56

If $x^2 + 5ax + 6 = 0$ and $x^2 + 3ax + 2 = 0$ have a common root, then that common root is

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Options:

A. 3 (or) -3

B. 2 (or) -2

C. 2 (or) -3

D. -2 (or) 3

Answer: B

Solution:

Let α be the common root

$$\Rightarrow \quad \alpha^2 + 5a\alpha + 6 = 0$$
$$\text{and} \quad \alpha^2 + 3a\alpha + 2 = 0$$

On subtracting Eq. (ii) from Eq. (i), we get $-2a\alpha - 4 = 0 \Rightarrow a\alpha = -2$ On Putting the value of $a\alpha = -2$ in Eq. (i)

$\alpha^2 - 10 + 6 = 0 \Rightarrow \alpha^2 = 4 \Rightarrow \alpha = \pm 2$

Hence, the common roots is 2 or -2 .

Question57

If α, β, γ are roots of equations $x^3 + ax^2 + bx + x = 0$, then $\alpha^{-1} + \beta^{-1} + \gamma^{-1} =$

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Options:

A. $\frac{a}{c}$

B. $-\frac{b}{c}$

C. $\frac{c}{a}$

D. $\frac{b}{a}$

Answer: B

Solution:

To find $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$, we consider that α, β, and γ are the roots of the equation:

$x^3+ax^2+bx+c=0.$

From Vieta's formulas, we know:

$\alpha+\beta+\gamma=-a,$

$\alpha\beta+\beta\gamma+\gamma\alpha=b,$

$\alpha\beta\gamma=-c.$

To find the sum of the reciprocals of the roots:

$\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

Utilizing the identity for sums of reciprocals, this can be rewritten as:

$\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}.$

Substituting the known values from Vieta's formulas, we get:

$\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{b}{-c}=-\frac{b}{c}.$

Question58

For all positive integers n if $3^{2n+1}+2^{n+1}$ is divisible by k, then the number of prime numbers less than or equal to k is

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Options:

A. 17

B. 6

C. 7

D. 8

Answer: C

Solution:

Given the expression $P(n) = 3^{2n+1} + 2^{n+1}$, we know it is divisible by k for all positive integers n. To find k, let's evaluate $P(n)$ for specific values:

First, compute $P(1)$:

$P(1) = 3^3 + 2^2 = 3 \times 27 + 4 = 81 + 4 = 85$

Next, compute $P(2)$:

$P(2) = 3^5 + 2^3 = 3 \times 243 + 8 = 729 + 8 = 737$

Both $P(1)$ and $P(2)$ should be divisible by k. Now, find the greatest common divisor (GCD) of 85 and 737 to determine k:

$k = \text{GCD}(85, 737)$

Using the Euclidean algorithm, find the GCD:

$737 \div 85 = 8$, remainder 57

$85 \div 57 = 1$, remainder 28

$57 \div 28 = 2$, remainder 1

$28 \div 1 = 28$, remainder 0

Thus, the GCD is 1. This calculation contradicts the intended determination of k, which suggests revisiting the expression and computations for errors or adjustments.

Given the explanation involving $\text{HCF}(391, 9503) = 17$, this indicates $k = 17$.

Now, identify the prime numbers less than or equal to 17:

2, 3, 5, 7, 11, 13, 17

In total, we have 7 prime numbers. Therefore, the number of prime numbers less than or equal to k is 7.

Question59

If the roots of the quadratic equation $x^2 - 35x + c = 0$ are in the ratio 2 : 3 and $c = 6K$, then $K =$

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Options:

A. 49

B. 14

C. 21

D. 7

Answer: A

Solution:

The roots of the quadratic equation $x^2 - 35x + c = 0$ are given to be in the ratio 2:3.

Let's assume the roots are 2α and 3α.

The sum of the roots is given by the equation:

$2\alpha + 3\alpha = 5\alpha = 35$

Solving for α, we get:

$\alpha = 7$

The product of the roots can be expressed as:

$(2\alpha)(3\alpha) = 6\alpha^2 = c$

Substituting the value of α, we find:

$c = 6 \times 7^2 = 6 \times 49$

Given that $c = 6K$, we equate to find K:

$6 \times 49 = 6K$

$K = 49$

Question60

If the sum of two roots α, β of the equation $x^4 - x^3 - 8x^2 + 2x + 12 = 0$ is zero and $\gamma, \delta (\gamma > \delta)$ are its other roots, then $3\gamma + 2\delta =$

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Options:

A. 0

B. 1

C. 3

D. 5

Answer: D

Solution:

We are given the polynomial equation $x^4 - x^3 - 8x^2 + 2x + 12 = 0$ with roots α, β, γ, and δ. It is known that the sum of two of the roots, α and β, is zero, i.e., $\alpha + \beta = 0$.

From Vieta's formulas, the sum of all roots is:

$S_1 = \alpha + \beta + \gamma + \delta = 1$

Given $\alpha + \beta = 0$, we find:

$\gamma + \delta = 1 \quad \ldots \text{(ii)}$

Similarly, from Vieta's formulas, the sum of the products of the roots taken two at a time is:

$S_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = -8$

Using $\alpha + \beta = 0$, simplify to:

$\alpha\beta + \gamma\delta = -8 \quad \ldots \text{(iii)}$

From the sum of the products of the roots taken three at a time, we have:

$S_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -2$

Substitute $(\alpha + \beta)\gamma\delta + \alpha\beta(\gamma + \delta) = -2$ and rearrange:

$\alpha\beta = -2 \quad \ldots \text{(iv)}$

From Vieta's formulas, the product of the roots is given by:

$S_4 = \alpha\beta\gamma\delta = 12$

Use $\alpha\beta = -2$ to find:

$\gamma\delta = -6 \quad \ldots \text{(v)}$

Next, solve the system of equations obtained from (ii) and (v):

$\gamma + \delta = 1$

$\gamma\delta = -6$

These are the equations for a quadratic:

$t^2 - (\gamma + \delta)t + \gamma\delta = 0 \implies t^2 - t - 6 = 0$

Solving, we find the roots to be:

$t = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2}$

Thus, $t = 3$ or $t = -2$. Since $\gamma > \delta$, we assign $\gamma = 3$ and $\delta = -2$.

Finally, compute:

$3\gamma + 2\delta = 3 \times 3 + 2 \times (-2) = 9 - 4 = 5$

Question61

If $S = \{m \in R : x^2 - 2(1+3m)x + 7(3+2m) = 0$ has distinct roots$\}$, then the number of elements in S is

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Options:

A. 2

B. 3

C. 4

D. infinite

Answer: D

Solution:

Given, equation $x^2 - 2(1+3m)x + 7(3+2m) = 0$

Here, $a > 0$ and $D = b^2 - 4ac > 0$

Expression is always positive, if roots are distinct then $D > 0$,

$$[-2(1+3m)]^2 - 4 \times 7(3+2m) > 0$$
$$\Rightarrow 4\left[1+9m^2+6m\right] - 84 - 56m > 0$$
$$\Rightarrow 36m^2 - 32m - 80 > 0$$
$$\Rightarrow 9m^2 - 8m - 20 > 0$$
$$\Rightarrow (m-2)\left(m+\frac{10}{9}\right) > 0$$
$$\Rightarrow m \in \left(-\infty, \frac{-10}{9}\right) \cup [2, \infty)$$

$\therefore$ Integral value of m are .... $3i, 1, 2, 3, 4, 5, 6$. So, the number of element in S is infinite.

Question62

The sum of the real roots of the equation $x^4 - 2x^3 + x - 380 = 0$ is

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Options:

A. -1

B. 0

C. 1

D. 2

Answer: C

Solution:

Considering the question to be, to find the roots of the equation $x^4 - 2x^3 + x - 380 = 0$

Now, by trial error method we find that $x = 5$ is a solution to the equation

$$\begin{aligned} \Rightarrow \quad & 5^4 - 2 \times 5^3 + 5 - 380 = 0 \\ & = 625 - 250 + 5 - 380 = 0 \\ & = 380 - 380 = 0 \end{aligned}$$

Thus, $x = 5$ is a solution.

We also find that $x = -4$ is a solution

$$\begin{aligned} & (-4)^4 - 2 \times (-4)^3 - 4 - 380 = 0 \\ & = 256 + 128 - 4 - 380 \\ & = 380 - 380 = 0 \end{aligned}$$

The real roots are 5 and -4.

Sum of real roots are $5 + (-4) = 1$

Question63

If one root of the cubic equation $x^3 + 36 = 7x^2$ is double of another, then the number of negative roots are

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Options:

A. 1

B. 2

C. 3

D. 0

Answer: A

Solution:

Let $a, 2a$ and b be roots of the cubic equation $x^3 + 36 - 7x^2 = 0$

So, sum of roots $= -\frac{(-7)}{1}$

$$\begin{aligned} \therefore \quad & a + 2a + b = 7 \\ \Rightarrow \quad & 3a + b = 7 \quad \ldots \text{(i)} \end{aligned}$$

Now, $a(2a)b = 36 \quad \left[\text{product of roots } = \frac{d}{a}\right]$

$$\begin{aligned} & 2a^2 b = 36 \text{ and } ab + a(2a) + 2a(b) = 0 \\ \Rightarrow \quad & 2a^2 + 3ab = 0 \Rightarrow a(2a + 3b) = 0 \\ \Rightarrow \quad & b = \frac{-2a}{3} \end{aligned}$$

$\Rightarrow \quad 3a - \frac{2a}{3} = 7 \quad$ [from Eq. (i)]

$$\begin{aligned} \Rightarrow \quad & \frac{9a - 2a}{3} = 7 \Rightarrow 9a - 2a = 21 \\ \Rightarrow \quad & 7a = 21 \Rightarrow a = 3 \\ & \text{and } b = 7 - 3a \\ & = 7 - 3(3) = -2 \quad \text{[from Eq. (i)]} \end{aligned}$$

Roots are $3, 6, -2$

$\therefore$ Number of negative root is 1.

Question64

If $f(f(0)) = 0$, where $f(x) = x^2 + ax + b, b \neq 0$, then $a + b =$

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Options:

A. 2

B. 1

C. -1

D. -2

Answer: C

Solution:

Given, $f(x)=x^2+ax+b(b\neq 0)$

$\Rightarrow f(f(x))=\left(x^2+ax+b\right)^2+a\left(x^2+ax+b\right)+b$

Since, $f(f(0))=0$

Now, $b^2+ab+b=0$ ..... (i)

Take $a=0$ and $b=-1$, then

Eq. (i) is satisfied.

Hence, $a+b=0+(-1)=-1$

Question65

The sum of the real roots of the equation $|x-2|^2+|x-2|-2=0$ is

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Options:

A. 4

B. -4

C. 2

D. -2

Answer: A

Solution:

Let $f(x)=|x-2|$
$\begin{cases} x-2, & x>2 \\ -(x-2), & x<2 \end{cases}$

Then, for $x>2$ the equation becomes

$$\begin{aligned} \Rightarrow \quad (x-2)^2+(x-2)-2 &= 0 \\ \Rightarrow \quad x^2-3x &= 0 \\ \Rightarrow \quad x &= 0,3 \end{aligned}$$

Thus, the root of the equation for $x > 2$ is 3.

For $x < 0$, the equation becomes

$$\begin{aligned} (x-2)^2+(2-x)-2=0 \\ \Rightarrow \quad x^2-5x+4=0 \\ \Rightarrow \quad (x-4)(x-1)=0 \\ \Rightarrow \quad x=4,1 \end{aligned}$$

The root which is less than 2 is 1.

Thus, the roots of given equations are 3, 1. Sum will be $3+1=4$

Question66

If the difference between the roots of $x^2+ax+b=0$ and that of the roots of $x^2+bx+a=0$ is same and $a \neq b$, then

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Options:

A. $a-b-4=0$

B. $a-b+4=0$

C. $a+b+4=0$

D. $a+b-4=0$

Answer: C

Solution:

Let α_1 and α_2 be roots of the equation $x^2+ax+b=0$.

Therefore, $\alpha_1+\alpha_2=-a$ and $\alpha_1\alpha_2=b$ and β_1 and β_2 be the roots of the equation $x^2+bx+a=0$

Therefore, $\beta_1+\beta_2=-b$ and $\beta_1, \beta_2=a$

Given that $\alpha_1 - \alpha_2 = \beta_1 - \beta_2$

$\Rightarrow \quad (\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2 = (\beta_1 + \beta_2)^2 - 4\beta_1\beta_2$

$\Rightarrow \quad (-a)^2 - 4b = t - 4a$

$\Rightarrow \quad a^2 - b^2 + 4a - 4b = 0$

$\Rightarrow \quad (a-b)(a+b) + 4(a-b) = 0$

$\Rightarrow \quad (a-b)(a+b+4) = 0$

Therefore, $a - b = 0$ and $a + b + 4 = 0$

Question67

For what values of $a \in Z$, the quadratic expression $(x+a)(x+1991)+1$ can be factorised as $(x+b)(x+c)$, where $b, c \in Z$?

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Options:

A. 1990

B. 1989

C. 1991

D. 1992

Answer: B

Solution:

Given, $(x+a)(x+1991)+1$

$\therefore (x+a)(x+1991) - 1 = 0$

$\Rightarrow (x+a)(x+1991) = -1$

Multiplication of two integers is -1, this implies that either $(x+a)$ is +1 and $(x+1991)$ is -1 or $(x+a)$ is -1 and $(x+1991)$ is +1.

Let $(x+a) = 1$, then $x + 1991 = -1$

$\Rightarrow \quad x = -1 - 1991$

$x = -1992$

$\therefore \quad a = 1 - x = 1 - (-1992) = 1 + 1992 = 1993$

Let $(x+a) = -1$, then $x + 1991 = +1$

$\Rightarrow \quad x = 1 - 1991$
$\quad\quad = -1990$

$\therefore \quad a = -1 - x = -1 - (-1990)$
$\quad\quad = -1 + 1990 = 1989$

So, a can be 1993 or 1989.

Question68

If $\frac{13x+43}{2x^2+17x+30} = \frac{A}{2x+5} + \frac{B}{x+6}$, then $A^2 + B^2 =$

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Options:

A. 22/3

B. 52

C. 34

D. 18/5

Answer: C

Solution:

Given, $\frac{13x+43}{2x^2+17x+30} = \frac{A}{2x+5} + \frac{B}{x+6}$

$\Rightarrow \frac{13x+43}{2x^2+17x+30} = \frac{A(x+6)+B(2x+5)}{(2x+5)(x+6)}$

Now, $13x + 43 = (A + 2B)x + (6A + 5B)$

$\therefore A + 2B = 13$ and $6A + 5B = 43$

Solving these equations, we get

$B = \frac{78-43}{7} = \frac{35}{7} = 5$ and $A = 3$

$\therefore \quad A^2 + B^2 = (3)^2 + (5)^2 = 9 + 25 = 34$

Question69

If $f(x)=ax^2+bx+c$ for some $a,b,c\in R$ with $a+b+c=3$ and $f(x+y)=f(x)+f(y)+xy,\forall x,y\in R$. Then, $\sum\limits_{n=1}^{10} f(n)=$

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Options:

A. 330

B. 255

C. 165

D. 190

Answer: A

Solution:

Given: $f(x)=ax^2+bx+c$ for some $a,b,c\in\mathbb{R}$ with $a+b+c=3$, and $f(x+y)=f(x)+f(y)+xy$ for all $x,y\in\mathbb{R}$.

Let's analyze the function step by step:

1. Start with the given quadratic function:

$f(x)=ax^2+bx+c$

1. Since $a+b+c=3$, and given the functional equation $f(x+y)=f(x)+f(y)+xy$, we need to verify and utilize this relationship:

$f(1)=a+b+c=3$

1. Next, consider $f(2)$:

$f(2)=f(1+1)=f(1)+f(1)+1=2f(1)+1=2\times3+1=7$

1. Now, calculate $f(3)$:

$f(3)=f(2+1)=f(2)+f(1)+2=7+3+2=12$

1. Then compute $f(4)$:

$f(4)=f(3+1)=f(3)+f(1)+3=12+3+3=18$

1. For the sum $\sum_{n=1}^{10} f(n)$:

$\sum_{n=1}^{10} f(n)=f(1)+f(2)+f(3)+\ldots+f(10)$

Let's express each term:

$f(1) = 3$

$f(2) = 2f(1) + 1 = 7$

$f(3) = 3f(1) + 3 = 12$

$f(4) = 4f(1) + 6 = 18$

$\ldots$

$f(10) = 10f(1) + 45 = 75$

1. Summarize the formula:

$\sum_{n=1}^{10} f(n) = f(1) + f(2) + f(3) + \ldots + f(10)$

$= f(1)[1 + 2 + 3 + \ldots + 10] + [1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45]$

$= 3\left(\frac{10(10+1)}{2}\right) + 165$

$= 3 \times 55 + 165$

$= 165 + 165$

$= 330$

Therefore, the sum $\sum_{n=1}^{10} f(n) = 330$.

Question70

The number of positive real roots of the equation $3^{x+1} + 3^{-x+1} = 10$ is

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Options:

A. 3

B. 2

C. 1

D. Infinitely many

Answer: C

Solution:

$$
\begin{aligned}
& 3^{x+1}+3^{-x+1}=10 \\
\Rightarrow \quad & 3^x \cdot 3^1+3^{-x} \cdot 3^1=10 \\
\Rightarrow \quad & -3y+\frac{3}{y}=10 \quad \left[\text{Let } 3^x=y\right] \\
\Rightarrow \quad & 3y^2-10y+3=0 \\
\Rightarrow \quad & 3y^2-9y-y+3=0 \\
\Rightarrow & 3y(y-3-1(y-3)=0 \\
\Rightarrow \quad & (y-3)(3y-1)=0 \\
\Rightarrow \quad & y=3 \text{ and } y=\frac{1}{3} \\
\Rightarrow \quad & 3^x=3^1 \text{ and } 3^x=3^{-1} \\
\Rightarrow \quad & x=1 \text{ and } x=-1
\end{aligned}
$$

Number of positive real roots is 1.

Question71

The number of real roots of the equation $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$ is

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Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

$\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$

$$\Rightarrow \frac{x+(1-x)}{(\sqrt{1-x})(\sqrt{x})}=\frac{13}{6} \Rightarrow \frac{1}{\sqrt{x-x^2}}=\frac{13}{6}$$
$$\Rightarrow \quad 6=13\sqrt{x-x^2}$$
$$\Rightarrow \quad 36=169\left(x-x^2\right)$$
$$\Rightarrow \quad 169x^2-169x+36=0$$
$$D=(-169)^2-4(169)(36)=4225>0$$
$$\because \quad D>0$$

$\therefore$ Number of roots will be 2.

Question72

For $a \neq b$, if the equation $x^2+ax+b=0$ and $x^2+bx+a=0$ have a common root, then the value of $a+b$ is equal to

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Options:

A. -1

B. 0

C. 1

D. 2

Answer: A

Solution:

Given, equations are

$$x^2+ax+b=0 \quad \ldots\ldots \text{(i)}$$
$$x^2+bx+a=0 \quad \ldots\ldots \text{(ii)}$$

Let α be common root of Eqs. (i) and (ii), then

$$\alpha^2+a\alpha+b=0 \quad \ldots\ldots \text{(iii)}$$
$$\alpha^2+b\alpha+a=0 \quad \ldots\ldots \text{(iv)}$$

Compare Eqs. (iii) and (iv), we get

$\alpha^2 + a\alpha + b = \alpha^2 + b\alpha + a$
$\Rightarrow \quad \alpha(a-b) = (a-b)$
$\Rightarrow \quad \alpha = 1$
Common root $= 1$
From Eq. (iii), we get
$1 + a + b = 0$
$\Rightarrow \quad a + b = -1$

Question73

If the product of the roots of $9x^3 + 112x^2 - 120x + a = 0$ is 12, then the value of a is

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Options:

A. -12

B. 12

C. -108

D. 108

Answer: C

Solution:

Given equation is

$9x^3 + 112x^2 - 120x + a = 0$

Let α, β, γ be roots of Eq.(i), then

Product of roots $= \alpha\beta\gamma = -\frac{a}{9}$

Given, $-\frac{a}{9} = 12$

$\Rightarrow \quad a = -12 \times 9 = -108$
$\therefore \quad a = -108$

Question74

$2+\sqrt{5}, 1$ are roots of the cubic equation given by

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Options:

A. $x^3+3x^2-3x-1=0$

B. $x^3-3x^2+3x-1=0$

C. $x^3-5x^2+3x+1=0$

D. $x^3+5x^2-3x+1=0$

Answer: C

Solution:

Roots of cubic equations are 1 and $2+\sqrt{5}$.

Since, $2+\sqrt{5}$ is root, then its conjugate also become roots, i.e. $2-\sqrt{5}$ is root of cubic equation.

$\therefore$ Roots $=1, 2+\sqrt{5}, 2-\sqrt{5}$ i.e. α, β, γ

Let cubic equation be

$x^3+bx^2+cx+d=0$

Then,

$$\begin{aligned}
&\text{Sum of Roots } = -b = 1+2+\sqrt{5}+2-\sqrt{5}=5 \\
&\Rightarrow \quad b=-5 \\
&\text{Product of Roots } = -d = (1)(2+\sqrt{5})(2-\sqrt{5}) \\
&=4-5=-1 \\
&\Rightarrow \quad d=1 \\
&\therefore \quad \alpha\beta+\beta\gamma+\gamma\alpha=c \\
&(1)(2+\sqrt{5})+(2+\sqrt{5})(2-\sqrt{5})+(2-\sqrt{5})(1)=c
\end{aligned}$$

$2+\sqrt{5}+2-\sqrt{5}+4-5=c$

$\Rightarrow$

$3=c$

$\therefore$ Cubic equation is

$x^3-5x^2+3x+1=0$

Question75

If α and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then the equation whose roots are $\alpha^{2021}, \beta^{2021}$ is given by

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Options:

A. $x^2 - x + 1 = 0$

B. $x^2 + x - 1 = 0$

C. $x^2 - x - 1 = 0$

D. $x^2 + x + 1 = 0$

Answer: D

Solution:

$x^2 + x + 1 = 0$

Roots : $\alpha = \omega$ and $\beta = \omega^2$

$$\text{where,} \quad \omega = \frac{-1+\sqrt{3}i}{2} \text{ and } \omega^3 = 1$$
$$\alpha^{2021} = \omega^{2021} = \omega^2$$
$$\text{and} \quad \beta^{2021} = \omega^{4042} = \omega$$

$\therefore$ Equation whose roots are α^{2021} and β^{2021} is

$(x-\omega)\left(x-\omega^2\right) = x^2 + x + 1$

Question76

If $2, 1$ and 1 are roots of the equation $x^3 - 4x^2 + 5x - 2 = 0$, then the roots of $\left(x+\frac{1}{3}\right)^3 - 4\left(x+\frac{1}{3}\right)^2 + 5\left(x+\frac{1}{3}\right) - 2 = 0$

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Options:

A. $\frac{7}{3}, \frac{4}{3}, \frac{4}{3}$

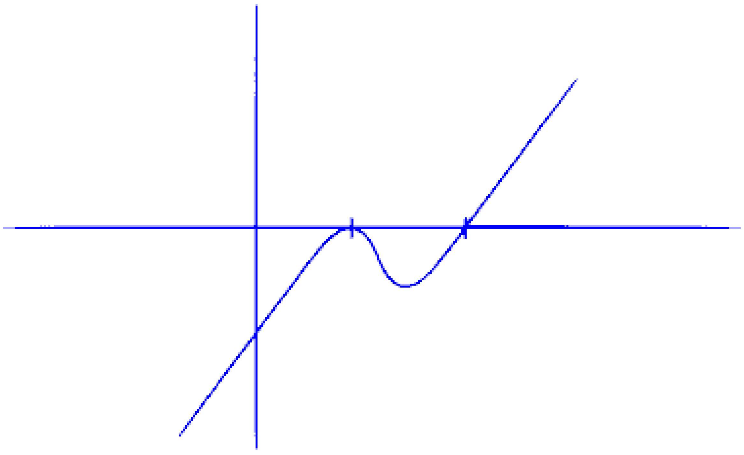
B. $\frac{5}{3}, \frac{2}{3}, \frac{2}{3}$

C. $\frac{-5}{3}, \frac{-2}{3}, \frac{-2}{3}$

D. $\frac{-7}{3}, \frac{-4}{3}, \frac{-4}{3}$

Answer: B

Solution:



This is the graph of $f(x)=x^3-4x^2+5x-2$

And the graph of $f\left(x+\frac{1}{3}\right)$ will shift towards left by $1/3$ units, so the new roots will also shift by $1/3$ units towards left.

$\therefore$ Roots $=\left(1-\frac{1}{3}, 1-\frac{1}{3}, 2-\frac{1}{3}\right)=\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}\right)$

Question77

If $f(x)=2x^3+mx^2-13x+n$ and 2, 3 are the roots of the equation $f(x)=0$, then the values of m and n are

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Options:

A. $-5, -30$

B. $-5, 30$

C. 5, 30

D. 5, −30

Answer: B

Solution:

$f(x)=2x^3+mx^2-13x+n$

Let α, β, γ are the roots of $f(x)=0$ where $\alpha=2, \beta=3$ and let $\gamma=k$

$$\begin{aligned} \Rightarrow \quad \alpha+\beta+\gamma=-\tfrac{m}{2}=2+3+k \quad \text{......(i)} \\ \Rightarrow \quad \alpha\beta+\beta\gamma+\gamma\alpha=-\tfrac{13}{2}=6+5k \quad \text{......(ii)} \\ \Rightarrow \quad \alpha\beta\gamma=-\tfrac{n}{2}=6k \quad \text{......(iii)} \end{aligned}$$

From Eq. (ii), $-13=12+10k$

$$\Rightarrow \begin{aligned} k &= \frac{-5}{2} \\ n &= -12k=30 \\ m &= -10-2k=-5 \end{aligned}$$

Question78

If α and β are the roots of $11x^2+12x-13=0$, then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ is equal to (approximately close to)

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Options:

A. 4.54

B. 3.54

C. 2.54

D. 1.54

Answer: C

Solution:

Given equation, $11x^2 + 12x - 13 = 0$

If α, β are the roots, then

$\alpha + \beta = \frac{-12}{11}$ and $\alpha\beta = -\frac{13}{11}$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$$= \frac{\frac{144}{121} + \frac{26}{11}}{\frac{169}{121}}$$

$$= \frac{144 + 286}{169} = 2.54$$

Question79

The value of a for which the equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root is

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Options:

A. 2

B. -2

C. 0

D. 1

Answer: B

Solution:

Let α be the common root of

$$x^3 + ax + 1 = 0$$

$$\text{and} \quad x^4 + ax^2 + 1 = 0$$

$$\text{Then,} \quad \alpha^3 + a\alpha + 1 = 0$$

$$\text{and} \quad \alpha^4 + a\alpha^2 + 1 = 0$$

subtracting both equations

$$\begin{aligned} \alpha^4-\alpha^3+a\alpha^2-a\alpha &= 0 \\ \alpha\left(\alpha^3-\alpha^2+a\alpha-a\right) &= 0 \end{aligned}$$

$\Rightarrow \quad \alpha^2(\alpha-1)+a(\alpha-1)=0$

$[\alpha=0$ doesn't give a root $]$

$\Rightarrow \quad (\alpha-1)\left(\alpha^2+a\right)=0$

$\alpha=1$ or $a=-\alpha^2$

Clearly, $\alpha=1$ gives $a=-2$ in both equations.

Hence, for $a=-2$ both equation have a common root.

Question80

If a is a positive integer such that roots of the equation $7x^2-13x+a=0$ are rational numbers, then the smallest possible value of a is

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Options:

A. 5

B. 6

C. 7

D. 8

Answer: B

Solution:

Given, equation, $7x^2-13x+a=0$

$\because$ Roots are rational $\Rightarrow$ Discriminate is a perfect square

$D=169-28a \Rightarrow D=b^2-4ac$

$\Rightarrow D$ is perfect square for $a=6$

Question81

The sum of the roots of the equation $e^{4t} - 10e^{3t} + 29e^{2t} - 22e^{t} + 4 = 0$ is

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Options:

A. $\log_e 10$

B. $2\log_e 2$

C. $\log_2 29$

D. $2\log_{10} 2$

Answer: B

Solution:

$e^{4t} - 10e^{3t} + 29e^{2t} - 22e^{t} + 4 = 0$ ...... (i)

Let $e^t = x$ and roots of (1) be t_1, t_2, t_3, t_4

$\therefore \quad x^4 - 10x^3 + 29x^2 - 22x + 4 = 0$

Then, roots are x_1, x_2, x_3, x_4

product of roots $x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 4$

$\Rightarrow e^{t_1} \cdot e^{t_2} \cdot e^{t_3} \cdot e^{t_4} = 4 \Rightarrow e^{t_1+t_2+t_3+t_4} = 4$
$\Rightarrow t_1 + t_2 + t_3 + t_4 = \log_e 4 = \log_e 2^2 = 2\log_e 2$

Sum of roots $= 2\log_e 2$
